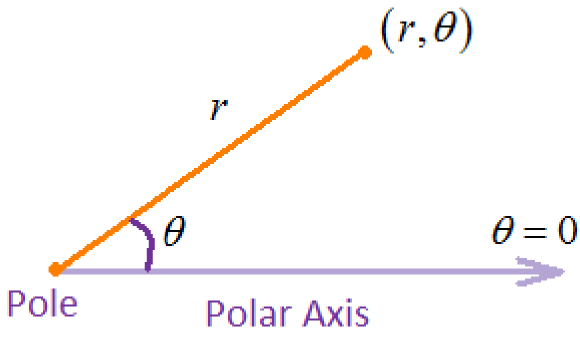
Polar Coordinates

For certain functions, rectangular coordinates (those using *x*-axis and *y*-axis) are very inconvenient. In rectangular coordinates, we describe points as being a certain distance along the *x*-axis and a certain distance along the *y*-axis.
But certain functions are very complicated if we use the rectangular coordinate system. Such functions may be much simpler in the **polar coordinate system**, which allows us to describe and graph certain functions in a very convenient way.

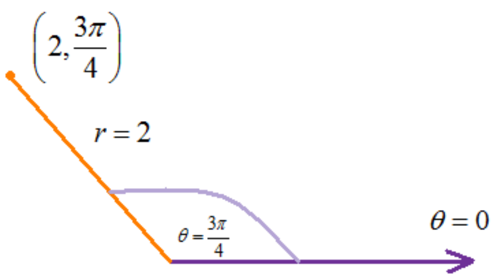
To form the polar coordinate system in the plane, fix a point O, called the pole (or origin), and construct from O an initial ray called the polar axis.

In polar coordinates, we describe points as being a certain **distance** (*r*) from the **pole** (the origin) and at a certain **angle** (θ) from the positive horizontal axis (called the **polar axis**).The coordinates of a point in polar coordinates are written as (r, θ)



r=directed distance from O to P
θ =directed angle, counterclockwise from polar axis to the segment

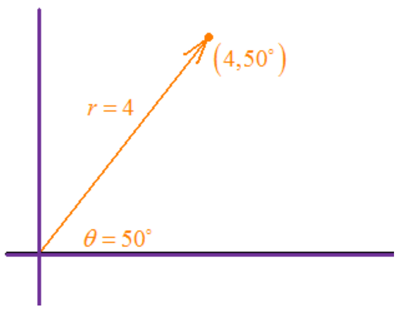
The point described in polar coordinates by (2, 3π/4) would look like this:



This diagram shows how the polar point (4,50°) is located:

Step 1: **Measure** the distance to a point 4 units from the origin along the positive ‘x’ axis.

Step 2: **Rotate** the point **anticlockwise** about the origin through angle 50°



Multiple Representations of a Point in Polar Coordinates

In rectangular coordinates, each point (x, y) has a unique representation. That is not true for polar coordinates. For instance, the coordinates (r,θ) and $(r,\theta+2\pi)$ represent the same point.

Another way to obtain multiple representations of a point is to use negative values for r. Because r is a directed distance, the coordinates (r,θ) and $(-r,\theta+\pi)$ represent the same point.

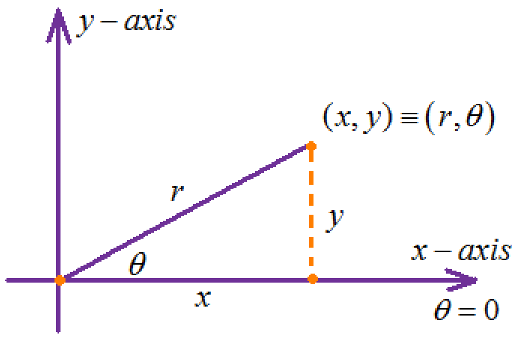
Rule 1: The point (r,θ) can be represented as:

$(r,\theta)=(r,\theta\pm 2n\pi)$ or $(r,\theta)=(-r,\theta\pm(2n+1)\pi)$ where n is any integer.

Remark: The pole is represented by $(r,\theta)=(0,\theta)$ where θ is any angle.

Coordinate Conversion

To establish the relation between polar and rectangular coordinates, let the polar axis coincide with the positive x-axis and the pole with the origin.
Because (x, y) lies on a circle of radius r, it follows that $r^2 = x^2 + y^2$. Moreover, for $r > 0$, the definitions of the trigonometric functions imply that:



- $\sin\theta = \dfrac{y}{r}$
- $\tan\theta = \dfrac{y}{x}$
- $\cos\theta = \dfrac{x}{r}$

Rule 2: The polar coordinates (r,θ) are related to the rectangular coordinates (x, y) as follows

Polar – to – Rectangular

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Rectangular – to – Polar

$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

So the **polar** point: (r,θ) can be converted **to rectangular** coordinates like this:

$$(r\cos\theta, r\sin\theta) \Rightarrow (x, y)$$

Example 1: A point has polar coordinates: $\left(5, \frac{\pi}{3}\right)$. Convert to rectangular coordinates.

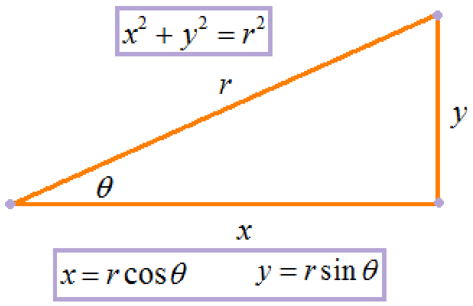
$$x = 5\cos\frac{\pi}{3} = 5\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{2}$$
$$y = 5\sin\frac{\pi}{3} = 5\left(\frac{1}{2}\right) = \frac{5}{2}$$
$$\left(5, \frac{\pi}{3}\right) = \left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$$

These formulas convert **rectangular** coordinates **to polar** coordinates:

By the rule of Pythagoras: $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \frac{y}{x}$

Therefore: $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. So the **rectangular** point: (x,y) can be converted **to polar** coordinates like this:

$$\left(\sqrt{x^2 + y^2}, \tan^{-1}\left(\frac{y}{x}\right)\right) \Rightarrow (r, \theta)$$



Equation Conversion

To convert an equation from polar or to polar we use the same method like the coordinates conversion. But when converting an equation from polar to rectangular form it is advisable to get rid of θ first.