Parametric Equations and Calculus

In physics, as well as many other branches of science, graphs of functions are simply inadequate to the task of modeling all of the phenomena which arise. In particular, to describe the motion of a particle in the *xy*-plane, it is necessary to use expressions that give the position *(x,y)* of the particle at any time *t*.

As a simple example, consider the trajectory of a ball that is thrown horizontally at a speed of 35 cm per second from a height of 5 cm above the ground by a person standing at the origin of coordinate system. In this coordinate system, *y* points in the vertical direction and *x* in the horizontal direction. If there is no air resistance, then there is nothing to increase the horizontal velocity of the ball. Therefore, *t* seconds after the ball is thrown, we expect the *x*-coordinate of the ball to be **35t** cm. The *y* coordinate will be governed by the constant acceleration model discussed in an earlier section, and *t* seconds after the ball is thrown we expect its height above the floor to be **5 - 16t²** cm. Thus, the path of the ball is well-expressed by the pair of equations:

$$\begin{cases} x(t) = 35t \\ y(t) = 5 - 16t^2 \end{cases}$$

A curve in the xy-plane may be described by a pair of parametric equations $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$

where x and y are related through their dependence on t. This is particularly useful when neither x nor y is a function of the other.

Definition 1: If f and g are continuous functions of t on an interval I, the set of ordered pairs $(f(t), g(t))$ **is a plane curve C. The equations**

$x = f(t)$ and $y = g(t)$

are parametric equations for C, and t is a parameter

<u>Changing from Parametric to Rectangular (Cartesian)</u>

To change a parametric equation back into a more familiar rectangular (Cartesian) equation you must eliminate the parameter. The typical approach to doing this is to solve for the parameter in one of the equations and then simply substitute that solution into the other equation.

Example 1: Eliminate the parameter from $x = 3t^2 - 4$ and $y = 2t$.

The y is definitely the easier function to solve for t

$$y = 2t \Rightarrow t = \frac{y}{2}$$

Substitute in the equation involving x, we get:

$$x = 3t^2 - 4$$

$$\Rightarrow x = 3\left(\frac{y}{2}\right)^2 - 4$$

$$\Rightarrow x = \frac{3y^2}{4} - 4$$