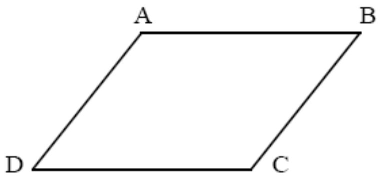
Parallelograms

A **parallelogram** is a quadrilateral with two pairs of parallel sides. A symbol for parallelogram *ABCD* is ▱ *ABCD*. In ▱ *ABCD* below, $\overline{AB}$ and $\overline{DC}$ are parallel sides. Also, $\overline{AD}$ and $\overline{BC}$ are parallel sides. The parallel sides are congruent.

Definition 1: A **parallelogram** is a quadrilateral whose opposite sides are parallel.

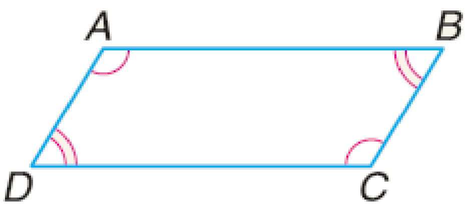


The defining property for a parallelogram is that it is a quadrilateral whose opposite sides are parallel. Many other properties follow from this. The most significant feature of the figure is that for either pair of opposite sides, the other two sides and the diagonals are transversals.

Thus, the theory of parallel lines and transversals may be used to prove properties of parallelograms. This theory and that for congruent triangles provide the needed tools for study of parallelograms.

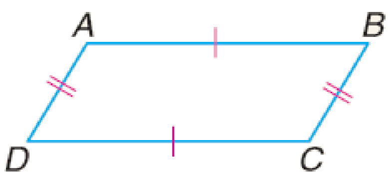
Theorem 1: Opposite angles of a parallelogram are congruent.

$\angle A \cong \angle C, \angle B \cong \angle D$



Theorem 2: Opposite sides of a parallelogram are congruent.

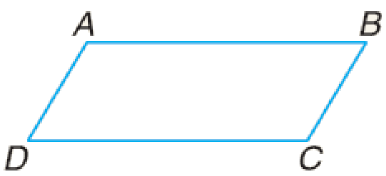
$\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$



Theorem 3: The consecutive angles of parallelogram are supplementary.

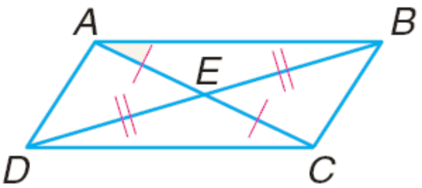
$m\angle A + m\angle B = 180^0$

$m\angle A + m\angle D = 180^0$



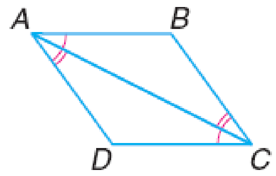
Theorem 4: The diagonals of a parallelogram bisect each other.

$\overline{AE} \cong \overline{EC}, \overline{BE} \cong \overline{ED}$



A diagonal separates a parallelogram into two triangles. You can use the properties of parallel lines to find the relationship between the two triangles. Consider ▱ $ABCD$ with diagonal $\overline{AC}$.

Example 1:



Given: ▱ $ABCD$

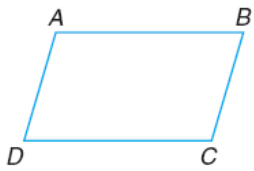
Prove: $\triangle ACD \cong \triangle CAB$

Proof:

Statements	Reasons
1) ▱ $ABCD$	1) Given
2) $\overline{DC} \parallel \overline{AB}$ and $\overline{AD} \parallel \overline{BC}$	2) Definition of parallelogram
3) $\angle ACD \cong \angle CAB$ and $\angle CAD \cong \angle ACB$	3) If two parallel lines are cut by a transversal, alternate interior angles are congruent.
4) $\overline{AC} \cong \overline{AC}$	4) Reflexive Property
5) $\triangle ACD \cong \triangle CAB$	5) ASA Postulate

Theorem 5: A diagonal of a parallelogram separates it into two congruent triangles.

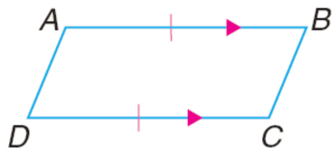
Theorem 2 states that the opposite sides of a parallelogram are congruent. Is the converse of this theorem true? In the figure, $\overline{AB}$ is congruent to $\overline{CD}$ and $\overline{AD}$ is congruent to $\overline{BC}$.



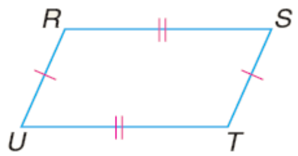
We know that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. If the opposite sides of a quadrilateral are congruent, then is it a parallelogram?

When is a quadrilateral a parallelogram?
To prove that a quadrilateral is a parallelogram we need to meet the requirements of either one of the theorems below:

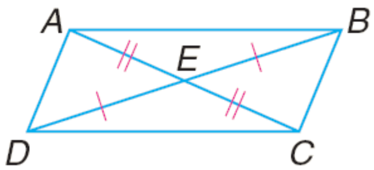
Theorem 6: If two sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



Theorem 7: If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.



Theorem 8: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



Theorem 9: If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.