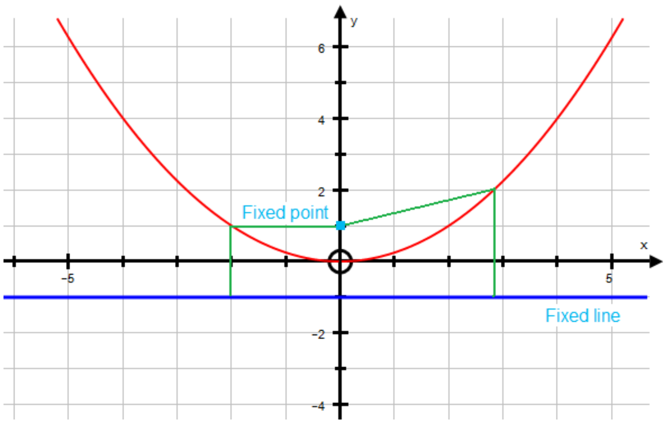
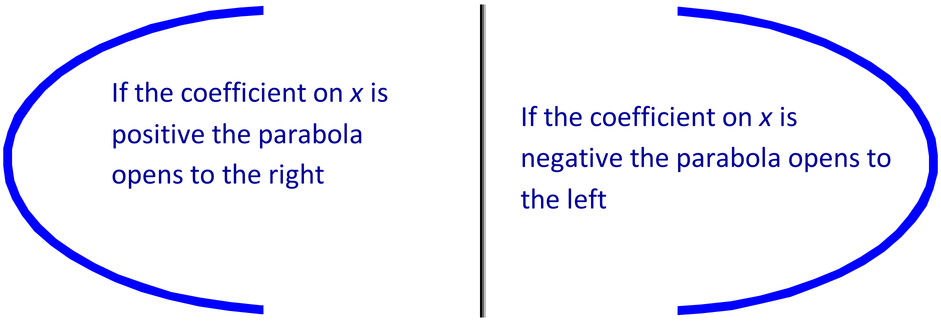
Parabolas

Definition 1: A quadratic function is in the form f(x) = $ax^2 + bx + c$. The graph of a quadratic function is a special type of U-shaped curve that is called a parabola.
A parabola can open either upward or downward, to the left or to the right.



A parabola is the set of all points P in the plane that are equidistant from a fixed point F (focus) and a fixed line d (directrix).

Based on this definition and using the distance formula we can get a formula for the equation of a parabola with a vertex at the origin that opens left or right: $y^2 = 4ax$, *where a* is the distance from the vertex to the focus (or opposite way for directrix)



<u>The equations of the parabola having center (0, 0) are as follows:</u>

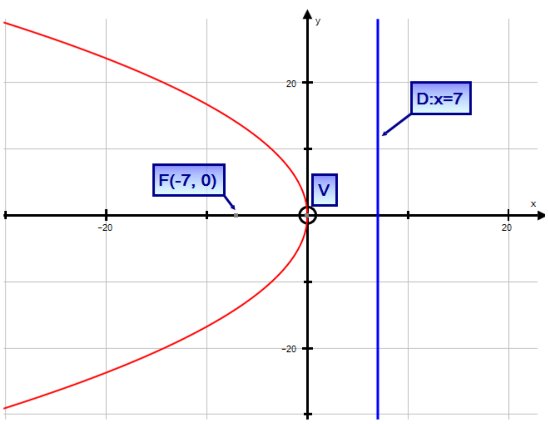
$x^2 = 4ay$ **Parabola opens up**

$x^2 = -4ay$ **Parabola opens down**

$y^2 = 4ax$ **Parabola opens right**

$y^2 = -4ax$ **Parabola opens left**

Example 1: Find the focus and directrix of the parabola $y^2 = -28x$



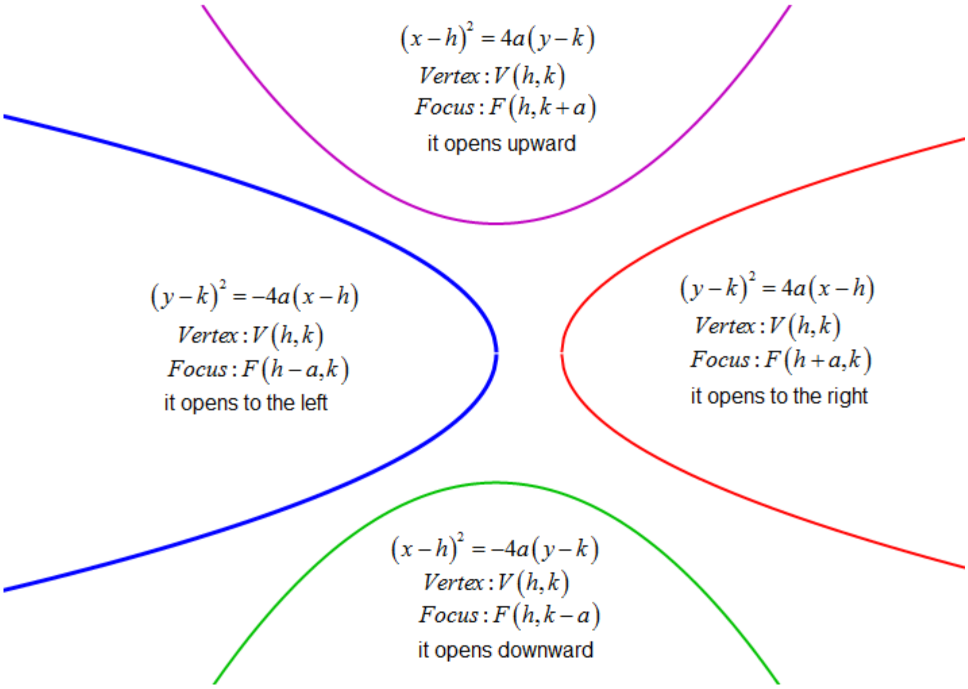
Since the coefficient of x is negative, this parabola opens to the left.

$$\left.\begin{array}{l} y^2 = -4ax \\ y^2 = -28x \end{array}\right\} \Rightarrow 4a = 28 \Rightarrow a = 7$$

Focus: F (-7, 0)
The equation of the directrix is: D: x=7
The equation of the axis of symmetry is: y=0

Translation of the vertex

If the vertex is transferred to a point of coordinates (h, k) then, the parabolas' equations will be different. The diagram shows the graph of each type of parabolas with its equation written in standard form.



Example 2: A parabola has a vertex at (-1,2) and passes through the point (1,-2). Find an equation to this parabola of the form $(x-h)^2 = 4a(y-k)$

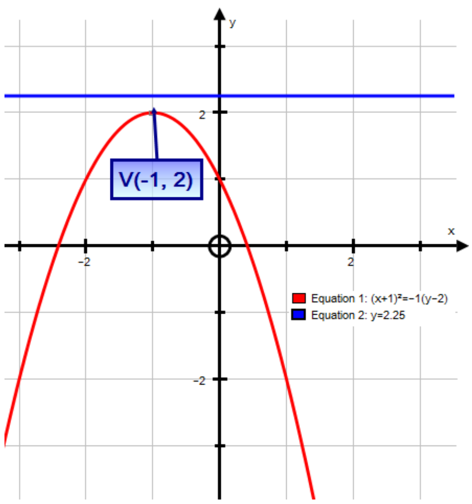
The x and y coordinates of the vertex gives the values of h and k respectively. Hence h = -1 and k = 2. The equation can be written as: $(x+1)^2 = 4a(y-2)$.

Now, we can find a using the fact that the point (1,-2) is on the graph of the line (substitute x by 1 and y by -2)

$$(1+1)^2 = 4a(-2-2)$$
$$\Rightarrow (2)^2 = 4a(-4)$$
$$\Rightarrow 4 = -16a$$
$$\Rightarrow a = \frac{4}{-16} = -\frac{1}{4}$$

The equation of the parabola can be written as

$$(x+1)^2 = -1(y-2)$$



A parabola has a vertex and a vertical axis of symmetry. It has the same graph on either side of a vertical line. The axis of symmetry passes through the vertex point and the focus point. The vertex point is either the maximum or minimum point for the parabola.

Example 3: Find the equation of the parabola with focus (1, 3) and directrix x = -3.

The parabola opens to the right. $\Rightarrow (y-k)^2 = 4a(x-h)$

The distance from the focus to the directrix is double the value of a

$\Rightarrow 2a = 1-(-3)$

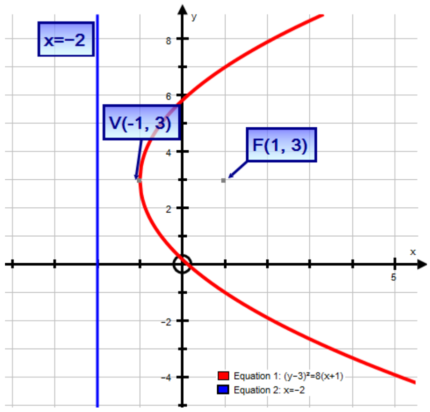
$\Rightarrow 2a = 4$

$\Rightarrow a = 2$

F is inside the parabola so $x_V = x_F - a = 1-2 = -1$ and

$y_V = y_F \Rightarrow V(-1,3)$

The equation is: $(y-3)^2 = 4(2)(x-(-1)) \Rightarrow (y-3)^2 = 8(x+1)$



Example 4: Find the standard equation of the parabola with vertex at the origin and focus (2,0).

The axis of the parabola is horizontal, passing through (0,0) and (2,0)

So, the standard form is $y^2 = 4px$, where $h=0$, $k = 0$, *and* $p = 2$, *so, the* equation *is* $y^2=8x$

Example 5: Find the focus of the parabola given by

$$y = -\frac{1}{2}x^2 - x + \frac{1}{2}$$

To find the focus, convert to standard from by completing the square.

$y = -\frac{1}{2}x^2 - x + \frac{1}{2}$	Write original equation.
$-2y = x^2 + 2x - 1$	Multiply each side by-2.
$1 - 2y = x^2 + 2x + 1$	Add 1 to each side.
$1 + 1 - 2y = x^2 + 2x + 1$	Complete the square.
$2 - 2y = x^2 + 2x + 1$	Combine like terms.
$-2(y-1) = (x+1)^2$	Standard form

Comparing this equation with

$(x-h)^2 = 4p(y-k)$

You can conclude that h=-1, k = 1, and p = $-\frac{1}{2}$. Because p is negative, the parabola opens downward. So, the focus of the parabola is $(h, k+p) = \left(-1, \frac{1}{2}\right)$.