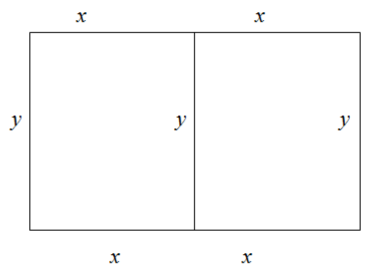
Optimization Problems

Optimization
If we wish to optimize a function, we are interested in finding its maximum or minimum.

APPLIED MINIMUM AND MAXIMUM PROBLEMS
Guidelines for Solving Applied Minimum and Maximum Problems
Step 1: Identify all given quantities and all quantities to be determined. MAKE A SKETCH!!!
Step 2: Write a **primary equation** for the quantity that is to be maximized or minimized.
Step 3: Reduce the primary equation to one having a single independent variable. You may need to use secondary equations relating the independent variables of the primary equation.
Step 4: Determine the **feasible** domain of the primary equation.
Step 5: Determine the desired maximum or minimum value using the techniques learned before.

MAXIMUM AREA

Example 1: A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?
Step 1:

GIVEN QUANTITIES	QUANTITIES TO BE DETERMINED
200 FEET OF FENCING  $P = 4x + 3y$	MAXIMUM AREA

Step 2: Write y in terms of x from the perimeter formula and find the general formula of the area

$$P = 4x + 3y$$
$$200 = 4x + 3y \quad \text{and} \quad A = 2xy$$
$$y = \frac{200 - 4x}{3}$$

Step 3: Write the area formula in terms of one variable (x) only

$$A = 2xy$$
$$A = 2x\left(\frac{200 - 4x}{3}\right)$$
$$A = \frac{400}{3}x - \frac{8}{3}x^2$$

Step 4: Domain: $[0, 200]$

Step 5: Find the derivative of A, then equate the derivative to zero to find the value of x

$$A=\frac{400}{3}x-\frac{8}{3}x^2$$

$$\frac{\delta A}{\delta x}=\frac{400}{3}-\frac{16}{3}x$$

$$0=\frac{400}{3}-\frac{16}{3}x$$

$$\frac{16}{3}x=\frac{400}{3}$$

$$x=\frac{400}{16}$$

$$x=25$$

$\frac{\delta^2 A}{\delta x^2}=-\frac{16}{3}<0$, by the second derivative test, we have the maximum area when $x=25\text{ ft}$ and $y=\frac{200-4x}{3}=\frac{200-4(25)}{3}=\frac{100}{3}\text{ft}$