

Operations with Polynomials

The use of letters in calculations is useful for solving problems as well as establishing some properties of numbers. We already know how to deal with an expression having letters and here we will study expressions having letters in a deeper way.

Definition 1: A **term** is a number, a variable, or a product of numbers and variables that have only positive exponents. A term or a monomial cannot have a variable as an exponent. When a term contains both a number and a variable part, the number part is called the "coefficient".

Definition 2: A monomial or the sum of one or more monomials is called a **polynomial**. **Poly-** a prefix meaning **many or several**; **Nomial-** means term, so polynomial means many terms

Definition 3: **Degree of a term** is the sum of the exponents on the variables in the term.

Definition 4: **Degree of a polynomial** is the degree of the highest degree term.

So to find the degree of a polynomial, find the degree of each term. The greatest of the degrees of its terms is the degree of the polynomial.

Arranging a polynomial: The terms of a polynomial are usually arranged so that the powers of **one variable** are in descending or ascending order.

To write a polynomial in **descending order** for a certain variable means to write the polynomial from the term with the highest exponent (in the certain variable) on the left descending to the term with the lowest exponent (in the certain variable) on the right.

Descending order in x : $3x^2 - 2x + 1$.

To write a polynomial in **ascending order** for a certain variable means to write the polynomial from the term with the lowest exponent (in the certain variable) on the left ascending to the term with the highest exponent (in the certain variable) on the right.

Ascending order in x : $1 - 2x + 3x^2$.

Note: A polynomial in standard form is in descending order.

Remark 2:

To evaluate a polynomial is like evaluating any algebraic expression. Steps to evaluate:

- 1) Replace the variable(s) in the expression with numeric values that are assigned to them
- 2) Simplify the resulting numeric expression by using the order of operation (PEMDAS)

Adding Polynomials

Definition 5: Monomials that have the same variable factors with the same degree are called **like terms**.

Rule 1: To add two polynomials, combine their like terms **by adding the numbers in front of the terms**.

There are different ways to add two polynomials:

- 1) Using algebra tiles
- 2) Using a horizontal method: Remove parentheses. Identify like terms. Group the like terms together. Add the like terms.
- 3) Using a vertical method: Arrange the like terms so that they are lined up under one another in vertical columns. Add the like terms in each column following the rules for adding signed numbers.

Subtracting Polynomials

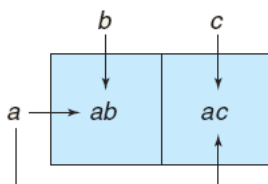
Rule 2: To subtract one polynomial from another, change the subtraction sign to an addition sign and change the signs of **all** the terms in the polynomial being subtracted (don't forget to change the sign of the first term). Then add the resulting polynomial by combining like terms.

Rule 3: Multiplying a polynomial by a monomial: To multiply a polynomial by a monomial, use the distributive property: multiply each term of the polynomial by the monomial. This involves multiplying coefficients and adding exponents of the appropriate variables.

Words: To multiply a polynomial by a monomial, use the Distributive Property.

Symbols: $a(b + c) = ab + ac$

Model:



Remark 1:

The number of terms of the product of a monomial by a polynomial (before simplification) is equal to the number of terms of the polynomial, i.e. (trinomial) \times (monomial) = (trinomial)

Multiplication of Binomials

To multiply a binomial by a binomial: $(a + b)(c + d)$ where $a, b, c,$ and d are terms--use the distributive property twice. First, treat the second binomial as a single term and distribute over the first binomial: $(a + b)(c + d) = a(c + d) + b(c + d)$

Next, use the distributive property over the second binomial: $a(c+d)+b(c+d)=ac+ad+bc+bd$

At this point, there should be 4 terms in the answer -- every term in the product consists of factors of the first binomial and a term of the second binomial.

Simplify the answer by combining like terms.

We can use the word **FOIL** to remember how to multiply two binomials $(a+b)(c+d)$:

Multiply their **F**irst terms. (ac)

Multiply their **O**utside terms. (ad)

Multiply their **I**nside terms. (bc)

Multiply their **L**ast terms. (bd)

Finally, add the results together: $ac+ad+bc+bd$. Combine like terms.

Multiplication of Binomials -- Special Cases

- I. **Square of a Binomial:** To square a binomial, multiply the binomial by itself:

$$(a+b)^2 = (a+b)(a+b)$$

$$(a-b)^2 = (a-b)(a-b)$$

$$(a+b)^2$$

$$= (a+b)(a+b)$$

$$= a(a+b) + b(a+b)$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a-b)^2$$

$$= (a-b)(a-b)$$

$$= a(a-b) - b(a-b)$$

$$= a^2 - ab - ab + b^2$$

$$= a^2 - 2ab + b^2$$

The square of a binomial is always the sum of:

- 1) The first term squared
- 2) 2 times the product of the first and second terms

The second term squared

Definition: When a binomial is squared, the resulting trinomial is called a **perfect square trinomial**.

- II. **Product of the Sum and Difference of Two Terms:** When we multiply two polynomials that are the sum and difference of the same 2 terms, $(x+5)$ and $(x-5)$ for example, we get an interesting result:

$$(a+b)(a-b)$$

$$= a(a-b) + b(a-b)$$

$$= a^2 - ab + ab - b^2$$

$$= a^2 - b^2$$

III. The cube of a binomial

There are similar formulas to find the cube of a binomial:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Multiplication of Polynomials

The strategy for multiplying two polynomials in general is similar to multiplying two binomials. First, treat the second polynomial as a single term, and distribute over the first polynomial:

$$(a + b + c)(x + y + z) = a(x + y + z) + b(x + y + z) + c(x + y + z)$$

Next, distribute over the second polynomial:

$$a(x + y + z) + b(x + y + z) + c(x + y + z)$$

$$= ax + ay + az + bx + by + bz + cx + cy + cz$$

Remark 2:

The number of terms in the answer (before simplification) should be the **number of terms** in the first polynomial times the **number of terms** in the second polynomial.

For example, if we are finding the product of two polynomials and there are 3 terms in each polynomial then the number of terms in the product is $3(3) = 9$.

If the first polynomial had 4 terms and the second had 5, there would be $4(5) = 20$ terms in the product before combining the like terms.