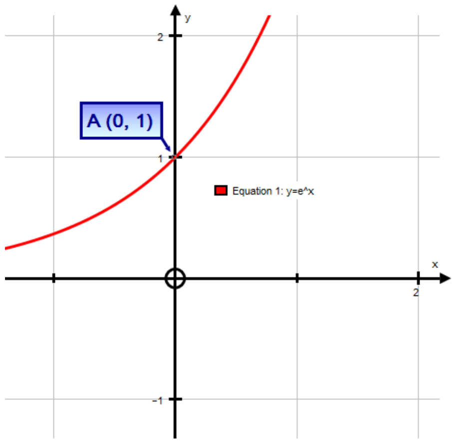
Natural Exponential & Logarithmic Functions

The Number e was first used by mathematician Leonard Euler to represent the base of natural logarithm in a letter to another mathematician, Christian Goldbach, in 1731.
In many applications the most convenient choice for a base is the irrational number $e \approx 2.718281828...$ this is called the natural base.
The function $f(x) = e^x$ is called the natural exponential function.



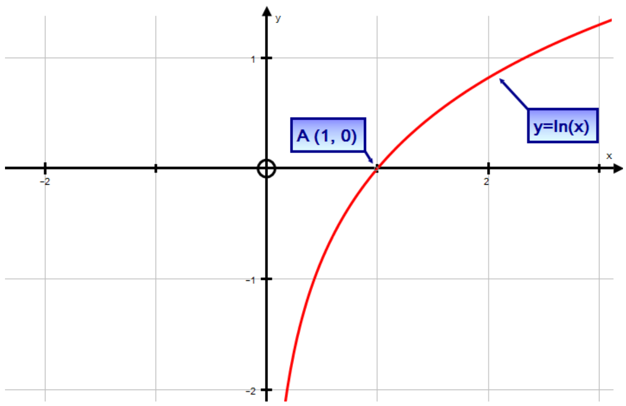
All the properties and characteristics of an exponential function are applicable on the function $f(x) = e^x$

Properties of $f(x) = e^x$: Let x and y be positive numbers

1) $e^0 = 1$
2) $e^x \bullet e^y = e^{x+y}$
3) $(e^x)^y = e^{x \bullet y}$
4) $\frac{e^x}{e^y} = e^{x-y}$
5) $\frac{1}{e^x} = e^{-x}$

Natural Logarithmic Function

The function $f(x) = \log_e x = \ln x$, $\underline{x > 0}$ is called the natural logarithmic function.



Every logarithmic function can be written in an equivalent exponential form and every exponential function can be written in an equivalent logarithmic form.

$y = \ln x \Leftrightarrow x = e^y$

Properties of Natural Logarithms

$\ln 1 = 0$ because $e^0 = 1$
$\ln e = 1$ because $e^1 = e$
$\ln e^x = x$ and $e^{\ln x} = x$ (Inverse Property)
$\ln x = \ln y \Leftrightarrow x = y$ (One – to – One Property)

Logarithmic form	Exponential Form
$y = \ln x$	$x = e^y$
$y = \log_a x$	$x = a^y$