Mutually Exclusive Events

Rule 1: The addition rule is a result used to determine the probability that event A or event B occurs or both occur. The result is often written as follows, using set notation:
P(*A* or *B*)= $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
where:
$P(A)$ = probability that event A occurs
$P(B)$ = probability that event B occurs
$P(A\cup B)$ = probability that event A or event B occurs
$P(A\cap B)$ = probability that event A and event B occur

Suppose we wish to find the probability of drawing either a king or a spade in a single draw from a pack of 52 playing cards.
We define the events A = "draw a king" and B = "draw a spade"
Since there are 4 kings in the pack and 13 spades, but 1 card is both a king and a spade, we have:

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$
$$=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}$$
$$=\frac{16}{52}$$

So, the probability of drawing either a king or a spade is: $\frac{16}{52}$

Rule 2: ***Mutually Exclusive Events:*** Two events are mutually exclusive (or disjoint) if it is impossible for them to occur together; they cannot happen at the same time.
Formally, two events A and B are mutually exclusive if and only if :

$$A\cap B=\varnothing \qquad \Rightarrow P(A\cap B)=0$$

If *A* and *B* are mutually exclusive events then the probability of *A* happening **OR** the probability of *B* happening is P(*A*) + P(*B*).

P(*A* or *B*)= $P(A\cup B)=P(A)+P(B)$

For example, if we toss a coin, either heads or tails might turn up, but not heads and tails at the same time.

Similarly, in a single throw of a die, we can only have one number shown at the top face. The numbers on the face are mutually exclusive events.

Example 1: Find the probability of having an even or an odd number when rolling a die.

Sample space S = {1,2,3,4,5,6}
Event A = "observe an odd number" = {1,3,5}
Event B = "observe an even number" = {2,4,6}
$A \cap B = \varnothing$ = the empty set, so A and B are mutually exclusive.

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\begin{aligned}
& P(AorB) \\
& = P(A) + P(B) \\
& = \frac{3}{6} + \frac{3}{6} \\
& = \frac{6}{6} \\
& = 1
\end{aligned}
$$

Note: Most of our statistics work is with mutually exclusive events: a variable has one value or another, but it can't have both. Either the population mean is greater than 62, or else it is less than or equal to 62.