Multiplying & Dividing Rational Expressions

Two rational expressions can be multiplied or divided just like two rational numbers.
You can use two methods to multiply rational numbers.

Method 1: Multiply numerators and multiply denominators. Then divide each numerator and denominator by the greatest common factor.

Method 2: Divide numerators and denominators by any common factors. Then multiply numerators and denominators.

Let us multiply $\frac{2x-3}{(x+1)^2}$ by $\frac{x+1}{2x-3}$ using the two methods stated above:

Method 1

Multiply, then simplify

$$\frac{2x-3}{(x+1)^2} \bullet \frac{x+1}{2x-3}$$
$$= \frac{(2x-3)(x+1)}{(x+1)^2(2x-3)}$$
$$= \frac{\cancel{(2x-3)}(x+1)}{(x+1)^2\cancel{(2x-3)}}$$
$$= \frac{1}{x+1}$$

Method 2

Simplify, then multiply

$$\frac{2x-3}{(x+1)^2} \bullet \frac{x+1}{2x-3}$$
$$= \frac{\cancel{2x-3}}{\cancel{(x+1)^2}} \bullet \frac{\cancel{x+1}}{\cancel{2x-3}}$$
$$= \frac{1}{x+1}$$

Rule 1: **Steps to multiply two rational expressions:**

Step 1: Cancel numerator to denominator if possible (don't cancel parts of a binomial or trinomial)

Step 2: Factor the numerators and denominators if possible.

Step 3: Multiply straight across - remember, you don't need a common denominator to multiply fractions (or rational expressions).

$$\frac{A}{B} \bullet \frac{M}{N} = \frac{A \bullet M}{B \bullet N} \qquad \textit{where} \qquad B \neq 0 \quad \textit{and} \quad N \neq 0$$

Example 1: Find each product

1) $\frac{12x}{5y} \bullet \frac{20y^2}{36x^2}$

$\frac{12x}{5y} \bullet \frac{20y^2}{36x^2}$ $x \neq 0; y \neq 0$

$= \frac{12\cancel{x}}{5\cancel{y}} \bullet \frac{20\cancel{y^2}^{\,y}}{36\cancel{x^2}_{\,x}}$

$= \frac{{}^{4}\cancel{240}\,y}{{}_{3}\cancel{180}\,x}$

$= \frac{4y}{3x}$

2) $\frac{a^2+6a+9}{a^2-9} \bullet \frac{a-3}{a-2}$

$\frac{a^2+6a+9}{a^2-9} \bullet \frac{a-3}{a-2}$ $a \neq \pm 3; a \neq 2$

$= \frac{(a+3)(a+3)}{(a-3)(a+3)} \bullet \frac{a-3}{a-2}$

$= \frac{(a+3)\,\cancel{(a+3)}}{\cancel{(a-3)}\,\cancel{(a+3)}} \bullet \frac{\cancel{(a-3)}}{(a-2)}$

$= \frac{a+3}{a-2}$

3) $\frac{3x-6}{5x-15} \bullet \frac{5x+25}{6x-12}$

$\frac{3x-6}{5x-15} \bullet \frac{5x+25}{6x-12}$ $x \neq 3; x \neq 2$

$= \frac{3\cancel{(x-2)}}{\cancel{5}\,(x-3)} \bullet \frac{\cancel{5}\,(x+5)}{6\cancel{(x-2)}}$

$= \frac{3(x+5)}{6(x-3)}$

$= \frac{x+5}{2x-6}$

Dividing rational expressions

To divide a rational number by any nonzero number, multiply by its reciprocal. You can use the same method to multiply rational expressions.

Rule 2: Steps to divide two rational expression

Step1: Change division to multiplication by inverting the divisor (second rational expression)
Step 2: Cross out the common factors (Do not attempt to cancel factors before it is written as a multiplication)
Step 3: Multiply the numerator by the numerator and the denominator by the denominator.

$$\frac{A}{B} \div \frac{M}{N} = \frac{A}{B} \bullet \frac{N}{M} = \frac{A \bullet N}{B \bullet M} \qquad where \qquad B \neq 0 \;; M \neq 0 \;\; and \;\; N \neq 0$$