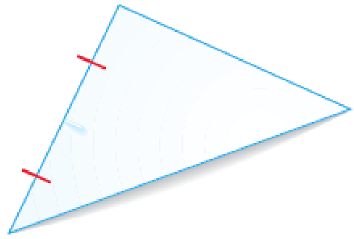
Mid-Segment Theorem

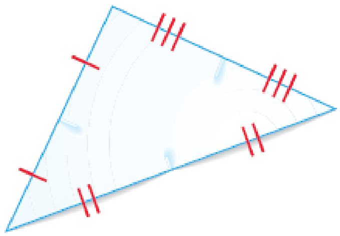
We studied four special types of segments of a triangle: perpendicular bisectors, angle bisectors, medians, and altitudes. Another special type of segment is called a *mid segment*. It is a segment that connects the midpoints of two sides of a triangle.

You can form the three mid segments of a triangle by tracing the triangle on paper, cutting it out, and folding it, as shown below.

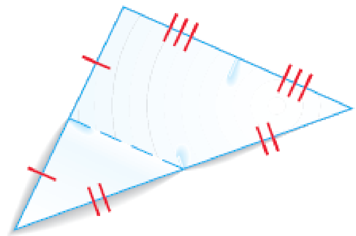
Fold one vertex onto another to find one midpoint.



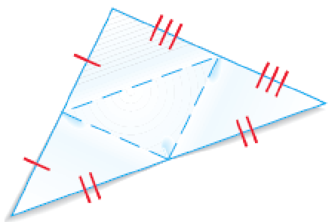
Repeat the process to find the other two midpoints.



Fold a segment that contains two of the midpoints.



Fold the remaining two mid segments of the triangle.



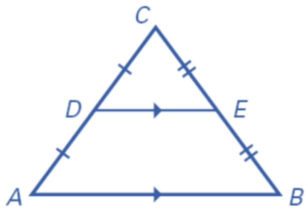
Definition 1: The **Mid Segment of a triangle** is a segment that connects the midpoints of two sides of a triangle.

Theorem 1: **Mid Segment Theorem**: the segment connecting the midpoints of two sides of a triangle is parallel to the third side and is equal to half its length.

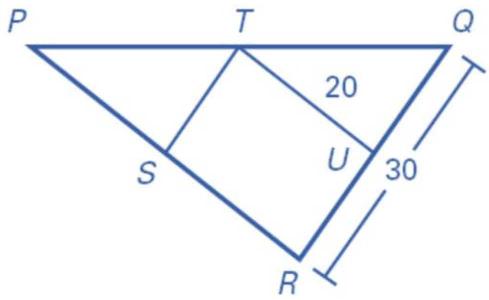
Given ΔABC with point D the midpoint of $\overline{AC}$ and point E the midpoint of $\overline{BC}$

DE ∥ AB

DE = $\frac{1}{2}$AB



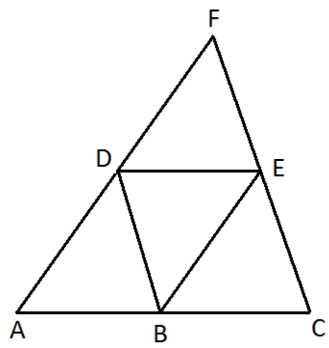
Example 1: $\overline{ST}$ and $\overline{TU}$ are mid segments of △*PQR*. Find *PR* and *ST*.



$$PR = 2(TU) = 2(20) = 40$$

$$ST = \frac{1}{2}(TS) = \frac{1}{2}(30) = 15$$

Example 2:



Given: D is the midpoint of $\overline{AF}$
E is the midpoint of $\overline{FC}$
B is the midpoint of $\overline{AC}$

Prove: $\triangle BDE \cong \triangle FDE$

Proof:

Statements	Reasons
D is the midpoint of $\overline{AF}$ B is the midpoint of $\overline{AC}$	Given
$BD = \frac{1}{2}FC = FE = EC$	Mid – Segments Theorem
$\overline{BD} \cong \overline{FE}$	Two segments of equal measures are congruent
E is the midpoint of $\overline{FC}$ B is the midpoint of $\overline{AC}$	Given
$EB = \frac{1}{2}AF = AD = DF$	Mid – Segments Theorem
$\overline{EB} \cong \overline{DF}$	Two segments of equal measures are congruent
$\overline{DE} \cong \overline{DE}$	Reflexive property
$\triangle BDE \cong \triangle FDE$	SSS theorem