Implicit Differentiation

An **explicit function** is a function whose dependent variable is completely defined in terms of its independent variable(s) in one equation. For example: if *y* is the dependent variable and $x_1, x_2, \ldots, x_n$ are the independent variables then $y = f(x_1, x_2, \ldots, x_n)$.

An **implicit function** is a function whose dependent variable cannot be completely defined in terms of its independent variable(s) in one equation. For example: if *y* is the dependent variable and $x_1, x_2, \ldots, x_n$ are the independent variables then $f(x_1, x_2, \ldots, x_n, y) = 0$.

To find the derivative $\frac{dy}{dx}$ of an implicit function requires us to use a process called **implicit differentiation**.

<u>**Use the following Steps to differentiate a function implicitly:**</u> $\frac{dy}{dx} = ?$

1) Simplify the equation if possible.

2) Differentiate both sides of the equation with respect to *x*. Use all the differentiation rules, be careful to use the **Chain Rule** when differentiating expressions involving *y*.
   a) Find $\frac{d}{dx}$ of each addend of the function containing only the variable *x*.
   b) For terms only containing the variable *y*, find $\frac{d}{dx}[f(y)] = \left[\frac{d}{dy}(f(y))\right] \cdot \frac{dy}{dx}$.
   c) For terms containing both *x* and *y* variables, use the product and chain rules.

3) Solve for $\frac{dy}{dx}$.

<u>**Note:**</u> It might be helpful to substitute *f* (*x*) into the equation for *y* before differentiating with respect to *x*. This will remind you when you must use the generalized forms of the **Chain Rule**. Since $f'(x) = \frac{dy}{dx}$, you differentiate with respect to *x* and substitute *y* for *f*(*x*) and $\frac{dy}{dx}$ for $f'(x)$. Then you can solve for $\frac{dy}{dx}$.

Example 1: Use implicit differentiation to find y'

$$x^2 + y^2 = 25$$
$$\Rightarrow x^2 + [f(x)]^2 = 25$$
$$\Rightarrow \frac{d}{dx}\left(x^2 + [f(x)]^2\right) = \frac{d}{dx}(25)$$
$$\Rightarrow 2x + 2[f(x)]f'(x) = 0$$
$$\Rightarrow f'(x) = \frac{-2x}{2[f(x)]}$$
$$\Rightarrow y' = \frac{dy}{dx} = \frac{-x}{y}$$