Imaginary and Complex Numbers

A positive number squared or a negative number squared will always equal to a positive number. We know how to calculate the square roots of positive numbers but what about the square roots of negative numbers. How can we find their values?

Mathematicians have designated a special number 'i' which is equal to the square root of - 1. Then, it follows that $i^2 = -1$. This new number is called "*i*", standing for "imaginary"

Definition 1: Imaginary number: $i^2 = -1 \Leftrightarrow i = \sqrt{-1}$

Example 1: Simplify

1. $\sqrt{-16}$

$$\sqrt{-16} = \sqrt{(-1)(16)} = \sqrt{(i^2)(4^2)} = \sqrt{(4i)^2} = 4i$$

2. $\sqrt{-144}$

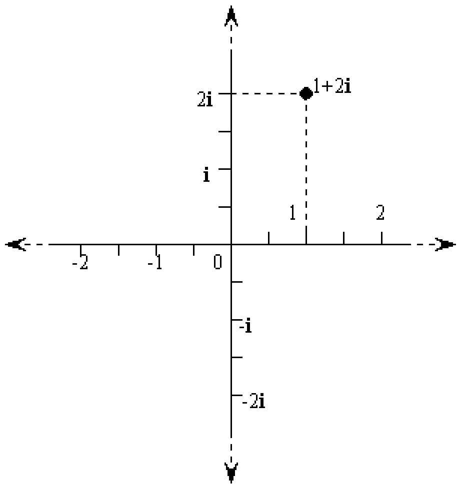
$$\sqrt{-144} = \sqrt{(-1)(144)} = \sqrt{(i^2)(12^2)} = \sqrt{(12i)^2} = 12i$$

Let us check the table to notice the relation between the results of even and odd powers of the imaginary number i.

$i = \sqrt{-1}$	$i^2 = -1$
$i^3 = i^2 \bullet i = (-1)i = -i$	$i^4 = i^2 \bullet i^2 = (-1)(-1) = 1$
$i^5 = i^2 \bullet i^2 \bullet i = (-1)(-1)i = i$	$i^6 = i^2 \bullet i^2 \bullet i^2 = (i^3)^2 = (-i)^2 = i^2 = -1$
$i^7 = (i^2)^3 \bullet i = (-1)^3 i = -i$	$i^8 = (i^2)^4 = (-1)^4 = 1$
$i^9 = (i^2)^4 \bullet i = (-1)^4 i = i$	$i^{10} = (i^2)^5 = (-1)^5 = -1$
$i^{11} = (i^2)^5 \bullet i = (-1)^5 i = -i$	$i^{12} = (i^2)^6 = (-1)^6 = 1$

Definition 2: A complex numbers have two parts, a "real" part (being any "real" number that you're used to dealing with) and an "imaginary" part (being any number with an "*i*" in it).
The "standard" form of a complex numbers is "$a+bi$"
Complex number: $z=a+bi$

Just as real numbers can be represented by points on the real number line, you can represent a complex number $z=a+bi$ as the point (a,b) in a coordinate plane (the complex plane).
The horizontal axis is called the real axis and the vertical axis is called the imaginary axis.
For example, the complex number $z=1+2i$ is represented by the point $(1,2)$



Definition 3: The absolute value of the complex number is the distance between the origin $(0,0)$ and the point (a,b).

$$|z| = |a+bi| = \sqrt{a^2+b^2}$$

Remark: If the complex number $a+bi$ is a real number (that is, b=0), then this definition agree with that given for the absolute value of a real number

$$|a+0i| = \sqrt{a^2+0^2} = \sqrt{a^2} = |a|$$