Higher Order Derivatives

So far, we have limited our discussion to calculating first derivative, $f'(x)$ of a function $f(x)$. What if we are asked to calculate higher order derivatives of $f(x)$.

A simple example of this is finding the acceleration of a body from a function that gives the location of the body as a function of time. The derivative of the location with respect to time is the velocity of the body, followed by the derivative of velocity with respect to time being the acceleration. Hence, the second derivative of the location function gives the acceleration function of the body.

Notation: Derivates can be denoted in several ways. The notations are

1st derivative (derivative of the original function $y = f(x)$): $f'(x),\ \frac{d}{dx}f(x),\ y',\ and\ \frac{dy}{dx}$

2nd derivative (derivative of the 1st derivative): $f''(x),\ \frac{d^2}{dx^2}f(x),\ y'',\ and\ \frac{d^2y}{dx^2}$

3rd derivative (derivative of the 2nd derivative): $\frac{d^3y}{dx^3} = f'''(x)$

For the n^{th} derivative, the notations are: $f^{(n)}(x),\ \frac{d^n}{dx^n}f(x),\ y^{(n)},\ \frac{d^ny}{dx^n}$

Distance functions

Suppose $s(t)$ is a distance function with respect to time *t*. Then $s'(t) = v(t)$ is an instantaneous velocity (or velocity) function with respect to time *t*, and $s''(t) = v'(t) = a(t)$ is an acceleration function with respect to time *t*.

Example 1: If $f(x) = x^2 \sin x$, then find $f'(x)$ and $f''(x)$.

First Derivative:

$$f'(x) = x^2 \cos x + 2x \sin x$$

Second Derivative:

$$f''(x) = x^2(-\sin x) + 2x\cos x + 2x\cos x + 2\sin x$$

$$f''(x) = -x^2 \sin x + 4x\cos x + 2\sin x$$