Name: _____

Higher Order Derivatives

1)   Let $y = \sin^{-1} x$. Find $\dfrac{d^2 y}{dx^2}$.

2)   Prove that $y = e^{-kx}(a\cos x + b\sin x)$ satisfies the equation $y'' + 2ky' + (k^2 + 1)y = 0$.

3)
     1)   Prove that if $y = x^2 \cos x$, then $x^2 \dfrac{d^2 y}{dx^2} - 4x \dfrac{dy}{dx} + (x^2 + 6)y = 0$.

     2)   Deduce that when $x = 0$, $(n-2)(n-3)\dfrac{d^n y}{dx^n} + n(n-1)\dfrac{d^{n-2} y}{dx^{n-2}} = 0$

4)   Given that $x = (t + 2)e^{3t}$:

1) Find:

     i) $\dfrac{dx}{dt}$

     ii) $\dfrac{d^2 x}{dt^2}$

2) Show that: $\dfrac{d^2 x}{dt^2} - 6\dfrac{dx}{dt} + 9x = 0$

5)   Given that $x = e^{kt}$, where k stands for some constant number:
1) Find:

     i) $\dfrac{dx}{dt}$

     ii) $\dfrac{d^2 x}{dt^2}$

2) If $\dfrac{d^2 x}{dt^2} - 2\dfrac{dx}{dt} - 3x = 0$ find the two possible values of k

6)   If $x = Ae^{3t} + Be^{-t}$, where A and B are standing for constant numbers, show that:
$\dfrac{d^2 x}{dt^2} - 2\dfrac{dx}{dt} - 3x = 0$

7) Find the second derivative of the function.

(1) $f(x) = 2x^2 + \sin 2x$

(2) $y = \tan x$

8) If $f(x) = x^2 + 2x - \dfrac{3}{x} + 5$, find $f^{(4)}(x)$

9) If $f(x) = x^2 + 2x - \dfrac{3}{x} + 5$, find $f^{(4)}(x)$.

10) A ball is thrown straight down from the top of a building. Its distance from the ground is given by the position function $s(t) = 220 - 22t - 16t^2$.

Find its velocity and acceleration after 3 seconds.