Graphs of Tangent and Cotangent Functions

Recall that the domain of the tangent function $y = \tan(x)$ consists of all numbers $x \neq (2n+1)\frac{\pi}{2}$; where n is any integer. The range consists of the interval $(-\infty, \infty)$.

Also, the tangent function is periodic of period π: Thus, we will sketch the graph on an interval of length π and then complete the whole graph by repetition. The interval we consider is the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ since this interval is of length π and the function is defined for any number inside this interval.

The tangent function is an odd function, i.e. $-\tan(x) = \tan(-x)$. Consequently, the graph of $y = \tan(x)$ is symmetric with respect to the origin.

First, we will consider the behavior of the tangent function to the right $-\frac{\pi}{2}$ and to the left of $\frac{\pi}{2}$ since the tangent function is not defined at these values. For this purpose, we construct the following table:

x	$-\frac{\pi}{2}$	-1.57	-1.5	-1.4	0	1.4	1.5	1.57	$\frac{\pi}{2}$
$\tan x$	*undefined*	-1255.77	-14.10	-5.80	0	5.80	14.10	1255.77	*undefined*

It follows that as x approaches $-\frac{\pi}{2}$ from the right the tangent function decreases without bound whereas it increases without bound when x gets closer to $\frac{\pi}{2}$ from the left.

We say that the vertical lines x = $\pm\frac{\pi}{2}$ are vertical asymptotes. In general, the **vertical asymptotes** for the graph of the tangent function consist of the zeros of the cosine function, i.e. the lines $x = (2n+1)\frac{\pi}{2}$; where n is an integer.

Next, we construct the following table that provides points on the graph of the tangent function:

x	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\tan x$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

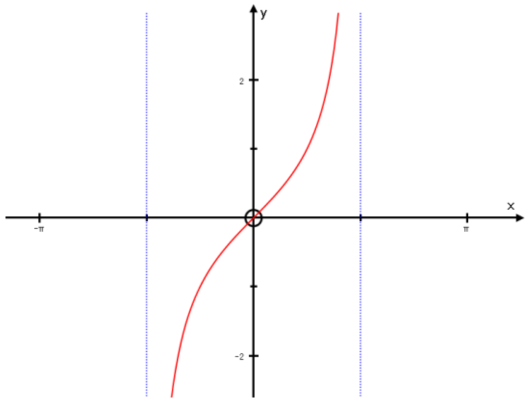
Plotting these points and connecting them with a smooth curve we obtain one period of the graph of y = tan x

Period: π

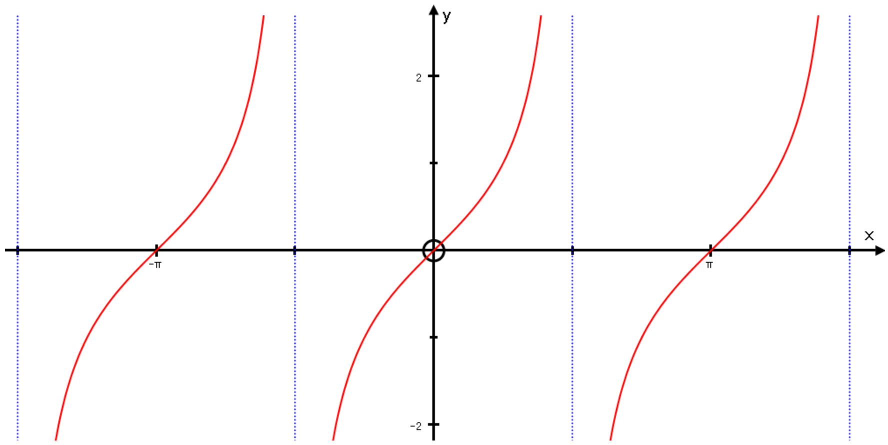
Domain: $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $x = \frac{\pi}{2} + n\pi$



We obtain the complete graph of $y = \tan x$ by repeating the one cycle over intervals of lengths π.



Example 1: What are the x-intercepts of $y = \tan x$?

The x-intercepts of y = tan x are the zeros of the sine function. That is, the numbers $x = n\pi$ where n is any integer.

Example 2: Sketch the graph of $y = \tan \frac{x}{2}$.

Asymptotes:

$$\frac{x}{2} = -\frac{\pi}{2} \Rightarrow x = -\pi$$

$$\frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$$

Therefore two consecutive asymptotes occur at $x = -\pi \,\&\, x = \pi$.

Period: $\frac{\pi}{b} = \frac{\pi}{\frac{1}{2}} = 2\pi$

Interval: $\frac{Period}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

Therefore the asymptotes are: $x = \pm\pi + 2\pi$

The functions $y = a\tan(bx)$ and $y = a\cot(bx)$; b > 0

1) The graphs of the tangent function and the cotangent function have no maximum or minimum, we conclude that these functions have **no amplitude**.
   The parameter $|a|$ indicates a vertical stretching of the basic tangent or cotangent function if a > 1; and a vertical compression if 0 < a < 1:
   If a < 0 then reflection about the x-axis is required.

2) $y = \tan x$ (respectively $y = \cot x$) completes one cycle on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (respectively, on $(0, \pi)$), the function $y = a\tan(bx)$ (respectively, $y = a\cot(bx)$) completes one cycle on the interval $\left(-\frac{\pi}{2b}, \frac{\pi}{2b}\right)$ (respectively, on the interval $\left(0, \frac{\pi}{b}\right)$ .Thus, these functions are periodic of **period** $\frac{\pi}{b}$

Rule: Guidelines for sketching graphs of tangent and cotangent functions

To graph $y = a\tan(bx - c) + d$ and $y = a\cot(bx - c) + d$; with b > 0; follow these steps:

Step 1: Find the period= $\frac{\pi}{b}$

Step 2: Find and Graph the asymptotes:

- $x = -\frac{\pi}{2b}$ and $x = \frac{\pi}{2b}$, for the tangent function.
- $x = 0$ and $x = \frac{\pi}{b}$ for the cotangent function.

For: $y = a\tan(bx - c) + d$ the asymptotes are: $bx - c = -\frac{\pi}{b}$ or $bx - c = \frac{\pi}{b}$

For: $y = a\cot(bx - c) + d$ the asymptotes are: $bx - c = 0$ or $bx - c = \pi$

Step 3: Divide the interval into four equal parts by means of the points:

- $-\frac{\pi}{4b}, 0, \frac{\pi}{4b}$ (for the tangent function).
- $\frac{\pi}{4b}, 0, \frac{3\pi}{4b}$ (for the cotangent function).

Step 4: Evaluate the function for each of the three x-values resulting from step 3.

Step 5: Plot the points found in step 4, and join them with a smooth curve.

Step 6: Draw additional cycles of the graph, to the right and to the left, as needed.