Graphs of Sine and Cosine Functions

Graphing is one of the most important tools available to scientists, engineers, and others in their efforts to understand functions. A graph characterizes a function's behavior and is an easily remembered visual image of its properties. Data analysis and problem solving frequently involves the use of graphs and the insights they provide. Consequently, a thorough knowledge of the graphs of the elementary trigonometric functions as well as an understanding of techniques used to graph more sophisticated trigonometric waves is instrumental to the student of trigonometry.
This lesson presents the basic graphing strategies used to graph generalized sine and cosine waves from a conceptual point of view.

The Sine Function: $y = \sin x$

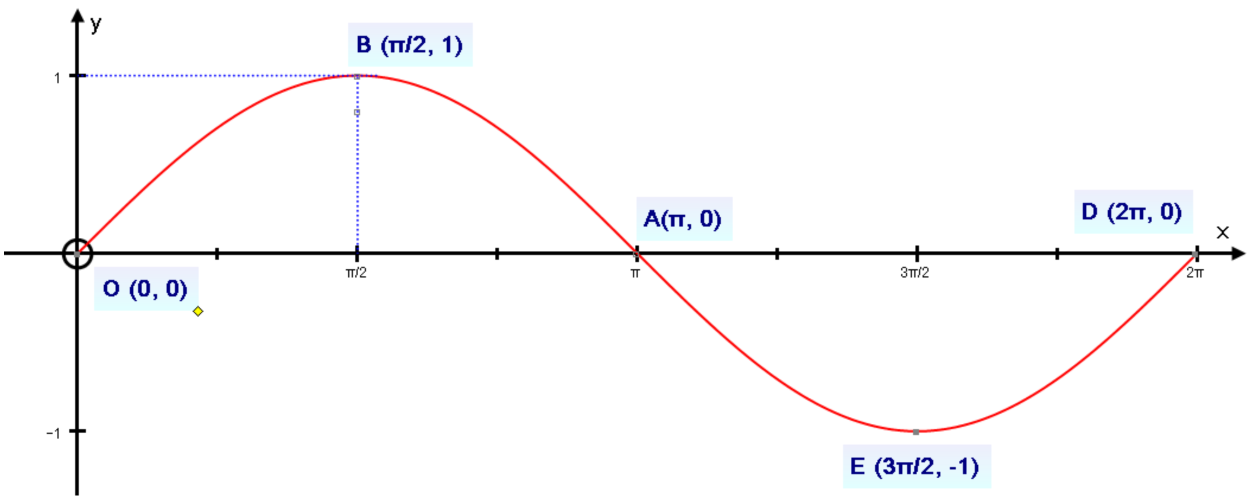
1- **Domain:** The domain of the sine function is the set of real numbers. To each x (think of x as an angle) corresponds a point on the unit circle. Its y − coordinate is $\sin x$.

2- **Range:** To each angle x corresponds a point on the unit circle. The coordinates of this point are (cos x, sin x). Also, the coordinates of a point on the unit circle are numbers between −1 and 1. This means that −1 ≤ sin θ ≤ 1. Thus, the range of sin x is [−1, 1].

3- **Amplitude:** is half the distance from the top of the curve to the bottom of the curve = 1

4- **Period:** The period of sin x is 2π. This means that this function repeats itself every interval of length 2π

x – value	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y – value	0	1	0	-1	0



Similar discussion should be made to graph $y = \cos x$, so we will obtain:

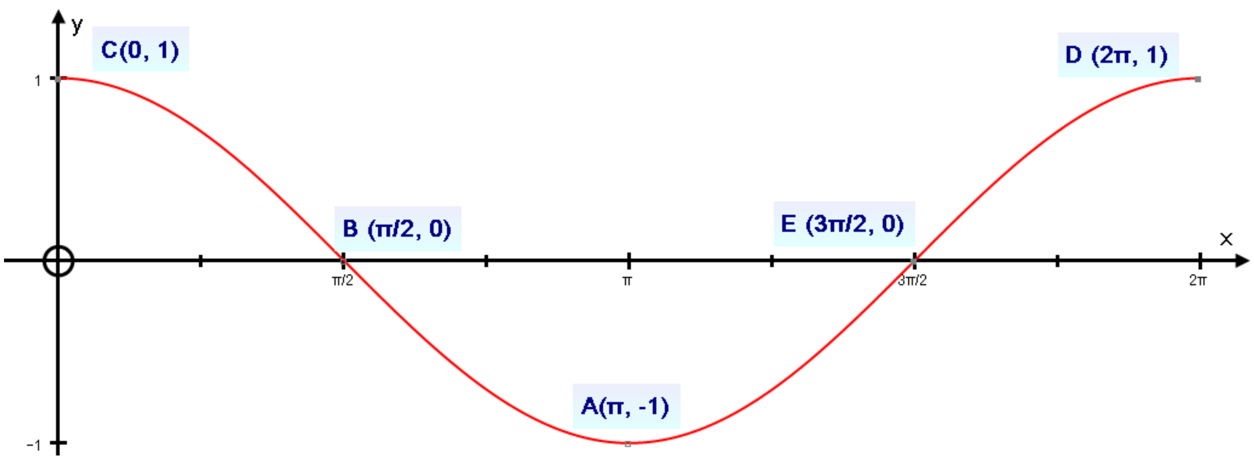
1- **Domain:** The domain of the $\cos$ine function is the set of real numbers. To each x (think of x as an angle) corresponds a point on the unit circle. Its y − coordinate is $\cos x$.

2- **Range:** To each angle x corresponds a point on the unit circle. The coordinates of this point are ($\cos x$, $\sin x$). Also, the coordinates of a point on the unit circle are numbers between −1 and 1. This means that $-1 \leq \cos\theta \leq 1$. Thus, the range of $\cos x$ is [−1, 1].

3- **Amplitude:** is half the distance from the top of the curve to the bottom of the curve = 1

4- **Period:** The period of $\cos x$ is 2π. This means that this function repeats itself every interval of length 2π

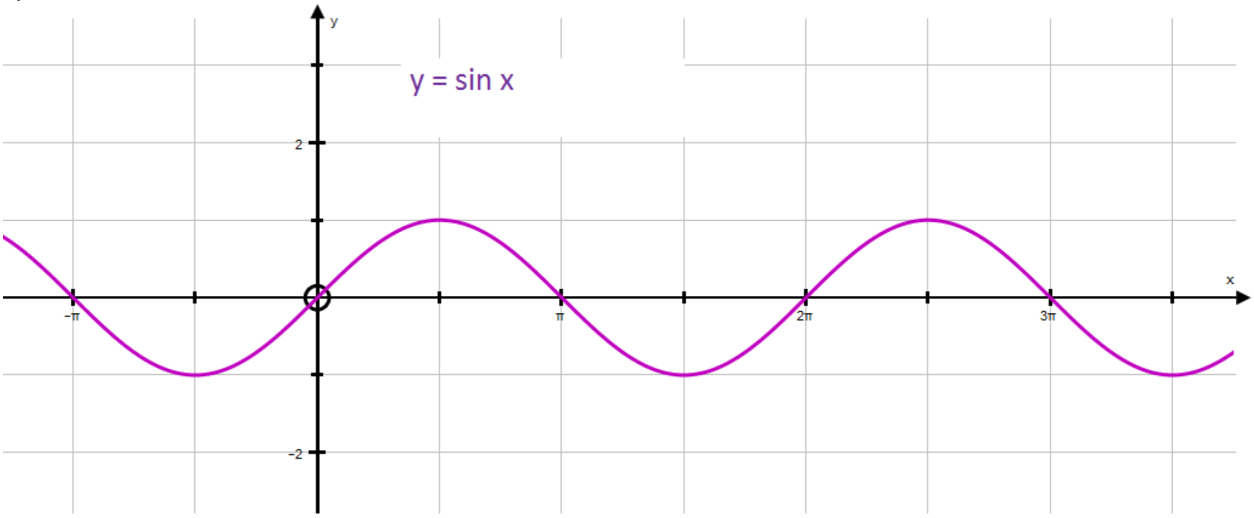
x – value	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y – value	1	0	-1	0	1

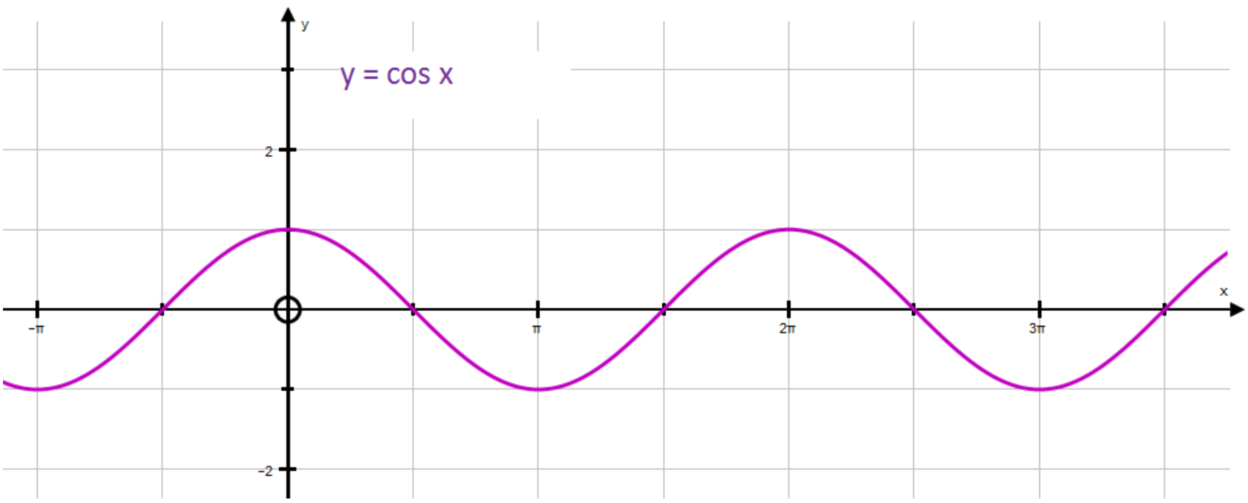


The table below shows the major steps that must be mentioned before graphing:

	$y = \sin x$	$y = \cos x$
Domain	□	□
Range	$[-1,1]$	$[-1,1]$
Period	2π	2π
Amplitude	1	1
Properties	$\sin(-x) = -\sin x$	$\cos(-x) = \cos x$

Adding or subtracting multiples of 360 to an angle does not alter the sin (or cos) of the angle. Hence the graph can be extended to the left and to the right by repeating cycles of the same shape every 360 as shown below:





Cycles are repeated every 360°. When using radian measure, cycles are repeated every 2π radians.

Definition 1: The amplitude of $y = a\sin x$ and $y = a\cos x$ represents half the distance between the maximum and the minimum values of the function and is given by:

$$Amplitude = |a|$$

Definition 2: The period of $y = a\sin bx$ and $y = a\cos bx$ is given by:

$$Period = \frac{2\pi}{b}$$

What about $y = 4\sin 3x$? or $y = 2\sin(3x+1)+5$? How to graph such functions?

In order to be able to graph such functions, we need to know what each number stands for, i.e., in $y = 2\sin(3x+1)+5$, the number 2 will cause a certain change, so as the 3, the 1 and the 5. The general form of the function of $y = \sin x$ is $y = a\sin(bx+c)+d; b>0$. We will discuss transformations of the sine function of the form $y = a\sin(bx+c)+d; b>0$.

Graphs of $y = a\sin(bx + c) + d; b > 0$

Let's look closely at the effects of each parameter a; b; c; and d:

1- The value of a .

This is outside the function and so deals with the output (i.e. the y values).
This constant will change the amplitude of the graph, or how tall the graph is. The amplitude, $|a|$; is half the distance from the top of the curve to the bottom of the curve. Multiplying the sine function by a results in a vertical stretch or compression (followed by a reflection about the x-axis if a < 0:)

2- The value of b .

This is inside the function so it affects the input or domain (i.e. the x values). This constant will stretch or compress the graph horizontally. However, it will not change the period directly. For example the function y = sin (2x) does not have period 2. The period is given by the fraction $\frac{2\pi}{b}$ (i.e. the original period divided by the constant b). So for example the function y = sin (2x) will have period $\frac{2\pi}{2} = \pi$ tells you the number of the cycles of the sine function on an interval of length 2 π: Thus, the graph of y = sin 2x consists of two cycles of the sine function on an interval like [0; 2 π]:

3- The value of d .

This again is outside and so will affect the y values of the graph. This constant will vertically shift the graph up and down (depending on d if positive or negative).

4- The constant c .

This is on the inside and deals with moving the function horizontally left/right.
For example the curve y = sin (x - 2) is the graph of y = sin (x) shifted horizontally to the right 2 units. Note that b = 1 in this example. For $b \neq 1$; the shift is $-\frac{c}{b}$. To see why this is so, recall that one cycle of y = a sin (bx + c) is completed for

$$0 \le bx + c \le 2\pi$$

Solving for x,we find:

$$\begin{aligned} &0 - c \le bx \le 2\pi - c \\ &\Rightarrow -c \le bx \le 2\pi - c \\ &\Rightarrow \frac{-c}{b} \le x \le \frac{-c}{b} + \frac{2\pi}{b} \end{aligned}$$

So, basically, the graph of $y = a\sin(bx + c)$ is shifted horizontally from the graph of $y = a\sin(bx)$ by $\frac{-c}{b}$ units. We call $\frac{-c}{b}$ the phase shift.

Guidelines for graphing $y = a\sin(bx+c)+d$, $b>0$

To sketch the graph of $y = a\sin(bx+c)+d$ follow these steps:

1- Find the domain
2- Find the range
3- Find the amplitude $= |a|$
4- Find the period $= \dfrac{2\pi}{b}$
5- Find the interval $= \dfrac{period}{4}$
6- Phase shift $= \dfrac{-c}{b}$
7- Vertical shift = d
8- Fill in the table below:

x – value	$\frac{-c}{b}$	$\frac{-c}{b}+\frac{\pi}{2b}$	$\frac{-c}{b}+\frac{\pi}{b}$	$\frac{-c}{b}+\frac{3\pi}{2b}$	$\frac{-c}{b}+\frac{2\pi}{b}$
y – value					

Example 1: Sketch the graph of $y = 2\sin(x-3)$

Compare $y = 2\sin(x-3)$ with $y = a\sin(bx+c)+d$ to define the parameters, $a=2, b=1, c=-3$

1- Domain: $\mathbb{R}$
2- Range: $-1 \le \sin(x-3) \le 1 \Rightarrow -2 \le 2\sin(x-3) \le 2 \Rightarrow y \in [-2,2]$
   Therefore, the range is $[-2,2]$
3- Amplitude $= |a| = |2|$
4- Period $= \dfrac{2\pi}{b} = \dfrac{2\pi}{1} = 2\pi$
5- Interval $= \dfrac{period}{4} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$
6- Phase shift $= \dfrac{-c}{b} = \dfrac{-(-3)}{1} = 3$

x – value	$\frac{-c}{b}=3$	$\frac{-c}{b}+\frac{\pi}{2b}$ $=3+\frac{\pi}{2}$	$\frac{-c}{b}+\frac{\pi}{b}$ $=3+\pi$	$\frac{-c}{b}+\frac{3\pi}{2b}$ $=3+\frac{3\pi}{2}$	$\frac{-c}{b}+\frac{2\pi}{b}$ $=3+2\pi$
y – value	$2\sin(0)=0$	$2\sin(\frac{\pi}{2})=2$	$2\sin(\pi)=0$	$2\sin(\frac{3\pi}{2})=-2$	$2\sin(2\pi)=0$

The points obtained from the table are: $(3,0);\left(3+\frac{\pi}{2},2\right);(3+\pi,0);\left(3+\frac{3\pi}{2},-2\right);(3+2\pi,0)$