Graph of Secant and Cosecant Functions

Graph of y=sec(x)

Recall that the domain of the secant function consists of all numbers $x \neq (2n+1)\frac{\pi}{2}$; where n is any integer. So the graph has vertical asymptotes at $x = (2n+1)\frac{\pi}{2}$

The range consists of the interval $(-\infty,1] \cup [1,\infty)$. Also, the secant function is periodic of period 2π: Thus, we will sketch the graph on an interval of length 2π and then complete the whole graph by repetition.

Note that the value of sec x at a given number x equals the reciprocal of the corresponding value of cos x: Thus, to sketch the graph of y = sec x; we first sketch the graph of y = cos x: On the same coordinate system, we plot, for each value of x, a point with height equal to the reciprocal of cos x: The accompanying table gives some points to plot.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$\sec x$	*undefined*	1.414	1	1.414	*undefined*	−1.414	−1	−1.414	*undefined*

Plotting these points and connecting them with a smooth curve we obtain the graph of y = sec x on the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \cup \left(\frac{\pi}{2},\frac{3\pi}{2}\right)$

The graph of y = sec x has vertical asymptotes at $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. It has minima (low points) at (0, 1) and (2π, 1). It has a maximum (high point) at $(\pi, -1)$.

Period: 2π

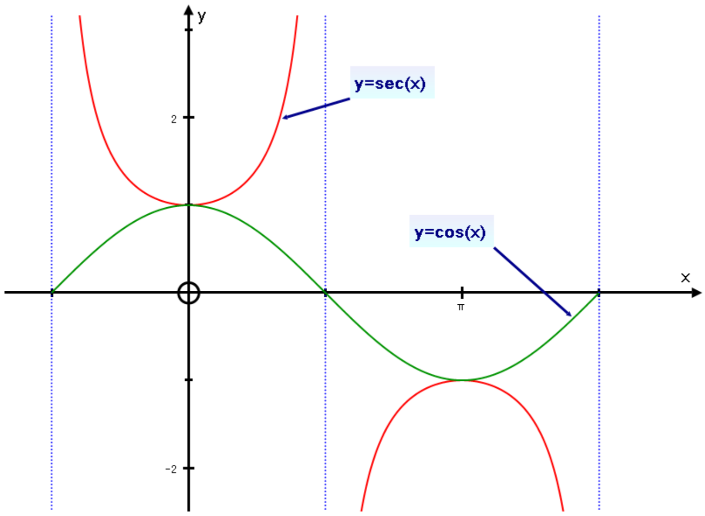
Domain: $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty,-1] \cup [1,\infty)$

Vertical Asymptotes:

$x = \frac{\pi}{2} + n\pi$

Symmetry: Origin



Example 1: What are the x-intercepts of y = sec x?

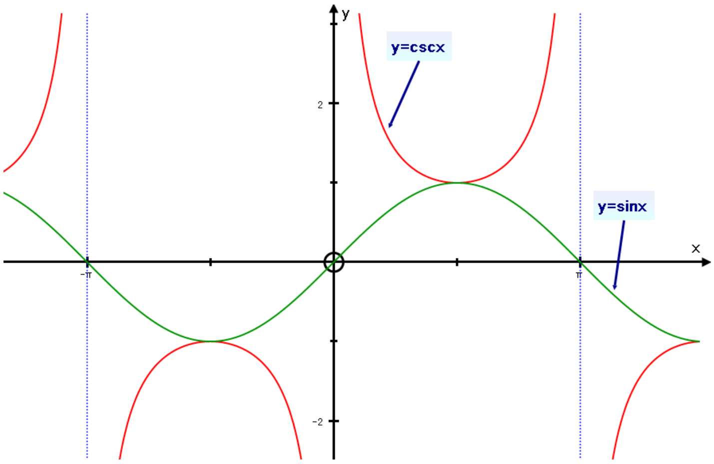
There are no x-intercepts since either $\sec x \geq 1$ & $\sec x \leq -1$

Graph of y = csc x

The graph of y = csc x may be graphed in a manner similar to sec x. Note that the vertical asymptotes occur at x = nπ; where n is an integer since the domain consists of all real numbers different from nπ:

Period: 2π
Domain: $x \neq n\pi$
Range: $(-\infty,-1]\cup[1,\infty)$
Vertical Asymptotes: $x = n\pi$

Symmetry: y-axis



Finally, note that in comparing the graphs of secant and cosecant functions with those of the sine and the cosine functions, the "hills" and "valleys" are interchanged. For example, a hill on the cosine curve corresponds to a valley on the secant curve and a valley corresponds to a hill.

Guidelines for Sketching Graphs of $y = a\sec(bx)$ and $y = a\csc(bx)$

To graph $y = a\sec(bx)$ or $y = a\csc(bx)$; with b > 0; follow these steps.

1. Find the period, $\frac{2\pi}{b}$

2. Graph the asymptotes:

$x = -\frac{\pi}{2b}$, $x = \frac{\pi}{2b}$ and $x = \frac{3\pi}{2b}$, for the secant function

$x = -\frac{\pi}{b}$, $x = 0$ and $x = \frac{\pi}{b}$ for the cosecant function

3. Divide the interval into four equal parts by means of the asymptotes and of the points:

0 , $\frac{\pi}{b}$ (For the secant function)

$-\frac{\pi}{2b}$, $\frac{\pi}{2b}$ (For the cosecant function)

4. Evaluate the function for each of the two x-values resulting from step

5. One of the points is the lowest of the "valley" and the other is the highest of the "hill."

6. Plot the points found in step 4, and join them with a smooth curve.

7. Draw additional cycles of the graph, to the right and to the left, as needed.