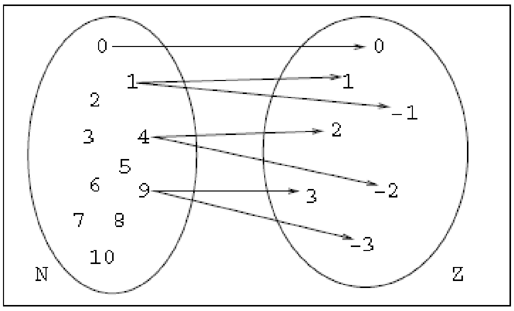
Functions

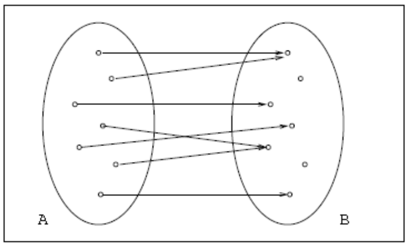
Some important sets are the following:
1. $\mathbb{N}$ = {0, 1, 2, 3, · · · } = the set of natural numbers.
2. $\mathbb{Z}$ = {· · ·,−3,−2,−1, 0, 1, 2, 3, · · · } = the set of integers.
3. $\mathbb{Q}$ = the set of rational numbers.
4. $\mathbb{R}$ = the set of real numbers.
5. $\mathbb{C}$ = the set of complex numbers.

Definition 1: Relation: Suppose that to each element of a set A we assign some elements of another set B. For instance, A = $\mathbb{N}$, B= $\mathbb{Z}$, and to each element $x \in \mathbb{N}$ we assign all elements $y \in \mathbb{Z}$ such that $y^2 = x$



This operation is called a **relation**

Definition 2: A function or mapping f from a set A to a set B, denoted $f : A \to B$, is a correspondence in which to each element x of A corresponds exactly one element y = f(x) of B.



Sometimes we represent the function with diagram like this:

$$f : A \to B \qquad A \xrightarrow{f} B$$
$$x \mapsto y \qquad x \mapsto y$$

For instance, the following represents the function from Z to Z defined by f(x) = 2x + 1:

$$f : \mathbb{Z} \to \mathbb{Z}$$
$$x \mapsto 2x+1$$

The element y = f(x) is called the image of x, and x is a pre-image of y. For instance, if f(x) = 2x + 1 then f(7) = 2 · 7 + 1 = 15. The set A is the domain of f, and B is its co-domain. The subset f(A) of B consisting of all images of elements of A is called the range of f. For instance, the range of f(x) = 2x + 1 is the set of all integers of the form 2x + 1 for some integer x, i.e., all odd numbers.

Two useful functions from □ to □ are the following:

Definition 3: The floor function: $\lfloor x \rfloor$ = greatest integer less than or equal to x

Example 1: $\lfloor 2 \rfloor = 2$; $\lfloor 2.3 \rfloor = 2$; $\lfloor \pi \rfloor = 3$; $\lfloor -2.5 \rfloor = -3$

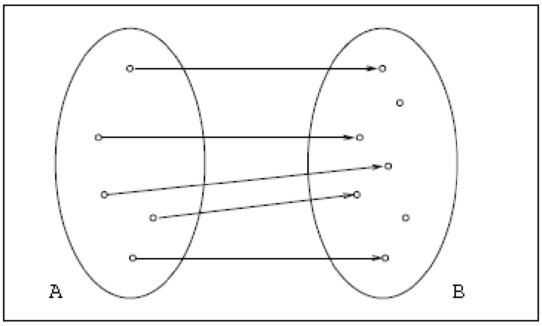
Definition 4: The ceiling function: $\lceil x \rceil$ = greatest integer less than or equal to x

Example 2: $\lceil 2 \rceil = 2$; $\lceil 2.3 \rceil = 3$; $\lceil \pi \rceil = 4$; $\lceil -2.5 \rceil = -2$

Types of Functions

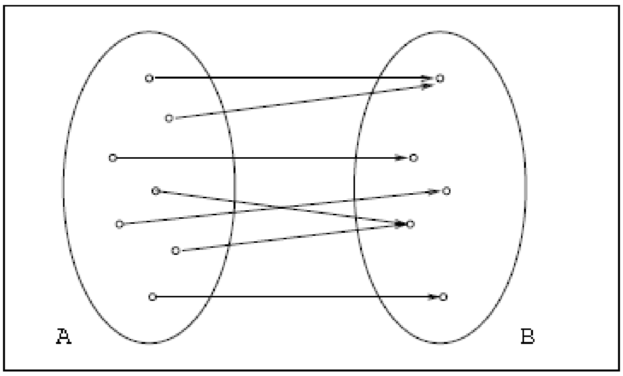
Definition 5: One – to – one or Injective: A function $f: A \to B$ is called one – to – one or injective if each element of B is the image of at most one element of A

$\forall a, b \in A, f(a) = f(b) \Rightarrow a = b$

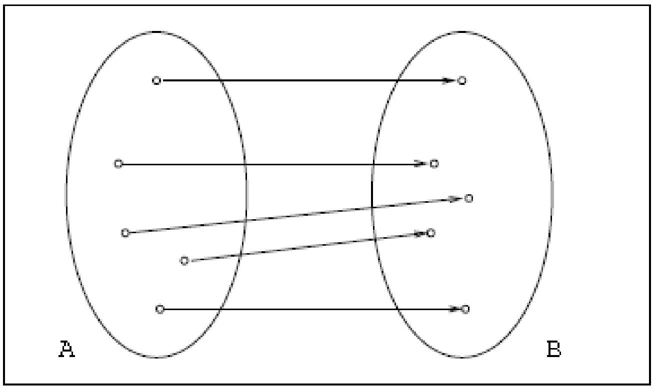


Definition 6: Onto or Surjective: A function $f: A \to B$ is called onto or Surjective if every element of B is the image of some element of A

$\forall y \in B, \exists x \in A \text{ such that } y = f(x)$

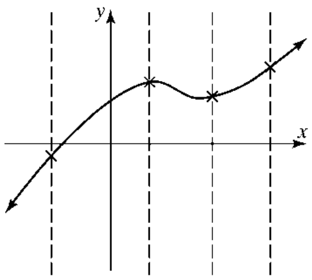


Definition 7: One – To – one Correspondence or Bijective: A function $f : A \to B$ is said to be a one – to – one correspondence, or Bijective, or a bijection, if it is one – to – one and onto at the same time.

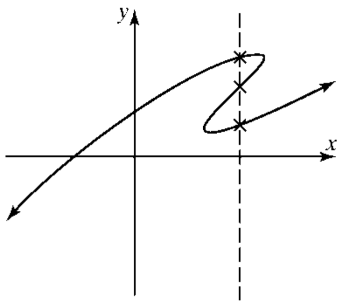


Identity Function: Given a set A, the function $1_A : A \to A$ defined by $1_A(x) = x$ for every x in A is called the identity function for A.

The Vertical Line Test: A graph in the Cartesian plane is the graph of a function if and only if no vertical line intersects the graph more than once.



This graph is a function.
(No vertical line intersects the graph more than once).



This graph is not a function.
(The graph does not pass the vertical line test).