Factors

Recall that when two or more numbers are multiplied, each number is a *factor* of the product. We all know that 3 • 6 = 18, 3 and 6 are called factors but 18 is called a product.

Some whole numbers have exactly two factors, the number itself and 1. Recall that these numbers are called ***prime numbers***. Whole numbers that have more than two factors, such as 12, are called ***composite numbers***. Check the table below for numbers below 20

Prime numbers less than 20	2, 3, 5, 7, 11, 13, 17, 19
Composite numbers less than 20	4, 6, 8, 9, 10, 12, 14, 15, 16, 18
Neither Prime nor Composite	0, 1

$4 \bullet 3 = 12$, 4 is a factor of 12. However, it is not a prime factor of 12 because 4 is not a prime number. Recall that when a number is expressed as a product of prime factors, the expression is called the ***prime factorization*** of the number.
The prime factorization of 12 is $2 \bullet 2 \bullet 3$

Definition 1: The greatest common factor of two (or more) integers is the largest integer that is a factor of both (or all) numbers.

Example 1: Consider the numbers 45, 15, and 90.
The greatest common factor is 15.Because 15 is the largest integer that will divide evenly into all three numbers.

Definition 2: The greatest common factor of two (or more) monomials is the product of the **greatest** common factor of the numerical coefficients (the numbers out in front) and the **least** power of every variable that is a factor of each monomial.

Definition 3: Two numbers or monomials whose greatest common factor is 1 are called **relatively prime**.

Rule 1: A polynomial is in *factored form* when it is expressed as the product of prime numbers and variables. To factor a polynomial we follow the steps below:

Step 1: Identify the GCF of the polynomial.
Step 2: Divide the GCF out of every term of the polynomial.

Remarks:

1. Be careful. **If a term of the polynomial is exactly the same as the GCF, when you divide it by the GCF you are left with 1, NOT 0.** Don't think, 'oh I have nothing left', there is actually a 1.

2. Note that if we multiply our answer out, we should get the original polynomial. In this case, it does check out. Factoring is **another way** to write the expression so it will be equivalent to the original problem.

Note: Not every polynomial is factorable. Just like not every number has a factor other than 1 or itself. A prime number is a number that has exactly two factors, 1 and itself. 2, 3, and 5 are examples of prime numbers.
The same thing can occur with polynomials.

Definition 4: **If a polynomial is not factorable we say that it is a prime polynomial.**

Sometimes you will not know that it is prime until you start looking for factors of it. Once you have exhausted all possibilities, then you can call it prime.

Be careful!! Do not think because you could not factor it on the first try that it is prime. You must go through ALL possibilities first before declaring it prime.