Factoring Special Products

In this lesson, we will first learn to factor trinomials that are squares of binomials. Then we factor trinomials that are differences of squares.

The trinomial has to be exactly in the same form to use the rules.

Part A: Trinomial Squares

How to Recognize a Trinomial Square

A) Two of the terms must be squares, such as A^2 and B^2.
B) There must be no minus sign before A^2 or B^2.
C) If we multiply A and B (which are the quantities that were squared) and double the result, we get the remaining term, 2AB, or its opposite, - 2AB.

When you have a base being squared plus or minus twice the product of the two bases plus another base squared, it factors as the sum (or difference) of the bases being squared.

Let us consider the polynomial $x^2 + 6x + 9$.
To factor it, we look for factors of 9 whose sum is 6. We see that these factors are 3 and 3, and the factorization is $x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$.

Example: Determine whether these are trinomial squares.

1) $x^2 + 10x + 25$
A) Two terms are squares: x^2 and 25.
B) There is no minus sign before either x^2 or 25.
C) If we multiply the quantities x and 5 and double the result, we get 10x, the remaining term.
Thus the trinomial, $x^2 + 10x + 25$, is a square.

2) $4x + 16 + 3x^2$
A) Only one term, 16, is a square. ($3x^2$ is not a square since 3 is not a perfect-square integer, and 4x is not a square since x is not a square.)
Therefore, the trinomial, $4x + 16 + 3x^2$, is not a square.

3) $100y^2 + 81 - 180y$
A) Two of the terms, $100y^2$ and 81, are squares.
B) There is no minus sign before either $100y^2$ or 81.
C) If we multiply the quantities 10y and 9 and double the product, we get the opposite of the remaining term: 2(10y)(9) = 180y, which is the opposite of - 180y.
Thus $100y^2 + 81 - 180y$ is a trinomial square.

To factor trinomial squares, we use the same equations that we used to multiply out squares of binomials. The identities are:

$$(a+b)^2 = a^2 + 2ab + b^2$$

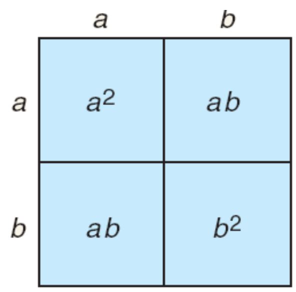
$$(a-b)^2 = a^2 - 2ab + b^2$$

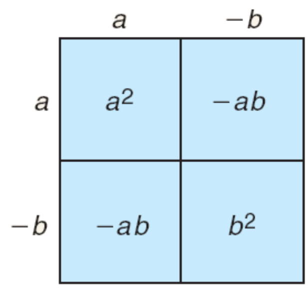
Rule 1: The steps below must be all satisfied to use the trinomial squares identities:

Step1: have a base being squared
Step2: plus another base squared
Step3: plus or minus twice the product of the two bases

It factors as the **sum** of the bases being squared if we have **plus** twice the product of the two bases.
It factors as the **difference** of the bases being squared if we have **minus** twice the product of the two bases.

The models of the identities are:



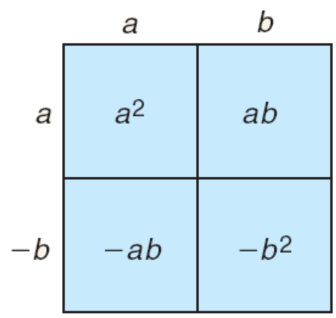


Part B: Difference of Squares

When you have the difference of two bases being squared, it factors as the product of the sum and difference of the bases that are being squared.
Although this is not a trinomial, it *can* be factored into two binomials.

$$a^2 - b^2 = (a-b)(a+b)$$

Check the model below:



Three conditions must be satisfied to use the identity:
1- Having a difference of terms
2- The first term is a perfect square of a number
3- The second term is a perfect square of a number.

Here is an example:

perfect squares

$$x^2 - 9 = \underbrace{(x-3)(x+3)}_{\text{product of a sum and a difference}}$$

difference

Remark:
An algebraic term is a perfect square when the numerical coefficient (the number in front of the variables) is a perfect square and the exponents of each of the variables are even numbers.

Remark:

It may be tempting to try to factor ($4x^2 + 9$). Notice that it is a sum of two expressions that are squares with no common factor. Such expressions cannot be factored using **real** numbers.

Be careful!! This process of factoring does NOT apply to $a^2 + b^2$.

Part C: Sum/Difference of Two Cubes

The Sum/Difference of Two Cubes method is used on cubic polynomials of the form: $a^3 \pm b^3$

By factoring an $a \pm b$ out of the expression we get:

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$