

Extrema on an Interval

Let f be defined on an open interval I containing c .

- $f(c)$ is the **minimum of f on I** if $f(c) \leq f(x)$ for all x in I .
- $f(c)$ is the **maximum of f on I** if $f(c) \geq f(x)$ for all x in I .

The minimum and maximum of a function on an interval are the **extreme values**, or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum** on the interval.

- ▶ A function does not need to have a maximum or minimum
- ▶ Extrema that occur at endpoints of an interval are called **endpoint extrema**

Theorem 1: THE EXTREME VALUE THEOREM: If f is continuous on a closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

RELATIVE EXTREMA AND CRITICAL NUMBERS

Definition 1:

- If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f , or you can say that f has a **relative maximum at $(c, f(c))$** .
- If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f , or you can say that f has a **relative minimum at $(c, f(c))$** .

Example 1: Find the value of the derivative (if it exists) at each indicated extremum.

$$1) f(x) = \cos \frac{\pi x}{2} ; (2, -1)$$

$$f'(x) = -\frac{\pi}{2} \sin \frac{\pi x}{2}$$

$$f'(2) = -\frac{\pi}{2} \sin \frac{\pi(2)}{2}$$

$$= -\frac{\pi}{2} \sin \pi$$

$$= -\frac{\pi}{2} \cdot 0$$

$$= 0$$

CRITICAL NUMBER

Definition 2: Let f be defined at c . If $f'(c)=0$, or if f is not differentiable at c , then c is a **critical number** of f .

Theorem 2: RELATIVE EXTREMA OCCUR ONLY AT CRITICAL NUMBERS

If f has a relative maximum or minimum at $x=c$, then c is a critical number of f .

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL

To find the extrema of a continuous function f on a closed interval $[a,b]$, use the following steps.

1. Find the critical numbers of f in (a,b) .
2. Evaluate f at each critical number in (a,b) .
3. Evaluate f at each endpoint of $[a,b]$.
4. The least of these numbers is the minimum. The greatest is the maximum.

the 1990s, the number of people in the UK who are aged 65 and over has increased from 10.5 million to 13.5 million (19.5% of the population).

There are a number of reasons why the number of people aged 65 and over has increased. One of the main reasons is that people are living longer. The life expectancy at birth in the UK is now 78 years for men and 82 years for women. This is a significant increase from the 1950s, when life expectancy at birth was 71 years for men and 76 years for women.

Another reason why the number of people aged 65 and over has increased is that people are having children later in life. This means that there are more people in the 65-74 age group than there were in the 1950s. This is because people who were born in the 1950s are now aged 65-74.

There are also a number of other factors that have contributed to the increase in the number of people aged 65 and over. These include the fact that people are working longer, the fact that people are getting married later in life, and the fact that people are having fewer children.

The increase in the number of people aged 65 and over has a number of implications for society. One of the main implications is that there is a need for more social care services. This is because people aged 65 and over are more likely to need help with everyday tasks, such as shopping, cooking, and cleaning.

Another implication is that there is a need for more housing for older people. This is because many older people live in overcrowded or unsuitable housing. This is often the case in inner city areas, where there is a high density of people and a high proportion of older people.

There are also a number of other implications of the increase in the number of people aged 65 and over. These include the fact that there is a need for more health care services, the fact that there is a need for more financial services, and the fact that there is a need for more leisure services.

The increase in the number of people aged 65 and over is a significant demographic change. It is a change that has a number of implications for society. It is a change that we need to be aware of and that we need to plan for.

There are a number of ways in which we can plan for the increase in the number of people aged 65 and over. One of the main ways is to invest in social care services. This is because social care services are essential for older people who need help with everyday tasks.

Another way to plan for the increase in the number of people aged 65 and over is to invest in housing for older people. This is because many older people live in overcrowded or unsuitable housing. Investing in housing for older people can help to improve their quality of life.

There are also a number of other ways to plan for the increase in the number of people aged 65 and over. These include investing in health care services, investing in financial services, and investing in leisure services.

The increase in the number of people aged 65 and over is a challenge for society. It is a challenge that we need to meet. We need to invest in social care services, housing for older people, health care services, financial services, and leisure services. We need to make sure that older people have the support they need to live well in old age.