Extrema on an Interval

Let f be defined on an open interval I containing c.

1. $f(c)$ is the **minimum of** f **on** I if $f(c) \leq f(x)$ for all x in I.
2. $f(c)$ is the **maximum of** f **on** I if $f(c) \geq f(x)$ for all x in I.

The minimum and maximum of a function on an interval are the **extreme values**, or **extrema** (the singular form of extrema is extremum), of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum** on the interval.

- A function does not need to have a maximum or minimum
- Extrema that occur at endpoints of an interval are called **endpoint extrema**

Theorem 1: THE EXTREME VALUE THEOREM: If f is continuous on a closed interval $[a,b]$, then f has both a minimum and a maximum on the interval.

RELATIVE EXTREMA AND CRITICAL NUMBERS

Definition 1:

1) If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f, or you can say that f has a **relative maximum at** $(c, f(c))$.
2) If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f, or you can say that f has a **relative minimum at** $(c, f(c))$.

Example 1: Find the value of the derivative (if it exists) at each indicated extremum.

1) $f(x) = \cos\frac{\pi x}{2}$; $(2, -1)$

$$f'(x) = -\frac{\pi}{2}\sin\frac{\pi x}{2}$$

$$f'(2) = -\frac{\pi}{2}\sin\frac{\pi(\cancel{2})}{\cancel{2}}$$

$$= -\frac{\pi}{2}\sin\pi$$

$$= -\frac{\pi}{2}\cdot 0$$

$$= 0$$

CRITICAL NUMBER

Definition 2: Let f be defined at c. If $f'(c)=0$, or if f is not differentiable at c, then c is a **critical number** of f.

Theorem 2: RELATIVE EXTREMA OCCUR ONLY AT CRITICAL NUMBERS
If f has a relative maximum or minimum at $x=c$, then c is a critical number of f.

GUIDELINES FOR FINDING EXTREMA ON A CLOSED INTERVAL
To find the extrema of a continuous function f on a closed interval $[a,b]$, use the following steps.

1. Find the critical numbers of f in (a,b).
2. Evaluate f at each critical number in (a,b).
3. Evaluate f at each endpoint of $[a,b]$.
4. The least of these numbers is the minimum. The greatest is the maximum.