Exponential and Logarithmic Differentiation

Let us derive the derivative of e^x using one of the basic properties of the natural logarithm.

Recall that $\ln e^x = x$.

$$\frac{d}{dx}\left[\ln e^x\right] = \frac{d}{dx}[x]$$

$$\frac{1}{e^x}\frac{d}{dx}\left[e^x\right] = 1$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

Let u be a differentiable function of x, the basic differential formulas for exponential and logarithmic functions are:

$$\frac{d}{dx}\left[e^u\right] = e^u \bullet \frac{du}{dx}$$

$$\frac{d}{dx}\left[a^x\right] = (\ln a)a^x \qquad \frac{d}{dx}\left[a^u\right] = (\ln a)a^u \bullet \frac{du}{dx}$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0 \qquad \frac{d}{dx}[\ln u] = \frac{1}{u} \bullet \frac{du}{dx}, \quad u > 0$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x} \qquad \frac{d}{dx}[\log_a u] = \frac{1}{(\ln a)u} \bullet \frac{du}{dx}$$

Example 1: Find the derivative of y.

1) $y = \mathrm{e}^{3x}$

$$y = \mathrm{e}^{3x} \Rightarrow \frac{dy}{dx} = \mathrm{e}^{3x} \times 3 = 3\mathrm{e}^{3x}$$

2) $y = 5^x$

$$y = 5^x \Rightarrow y' = (\ln 5)5^x$$

Note: **Be careful:** There are different types of functions. Each function uses a different differentiation formula, depending on whether the base and exponent are constants or variables.

Case 1: Variable raised to the constant power: $\frac{d}{dx}\left(x^3\right) = 3x^2$

Case 2: Constant raised to the variable power: $\frac{d}{dx}\left(3^x\right) = 3^x \ln 3$