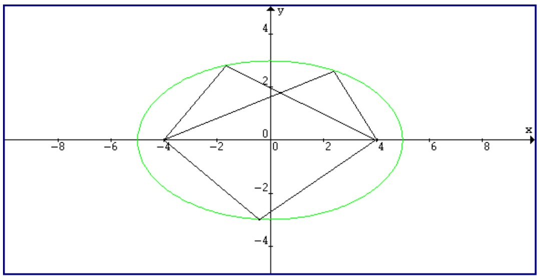
Ellipse



Definition: If $F_1(c,0)$ and $F_2(-c,0)$ are two fixed points in the plane and a is a constant, 0< c < a, then the set of all points P in the plane such that $PF_1 + PF_2 = 2a$ is an ellipse. F_1 and F_2 are the foci of the ellipse.

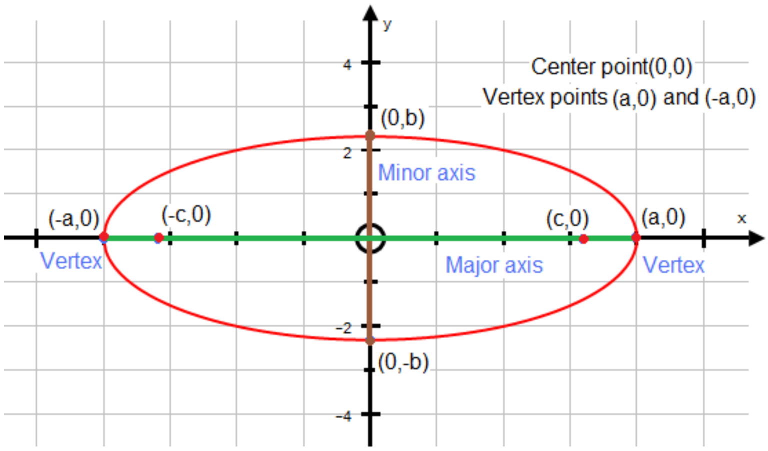
Notice the two fixed points in the graph, (-4, 0) and (4, 0). These are the foci points for the graph. (By definition, the distance from these points to a point on the ellipse is a constant.) In this case the constant is 10. No matter what point you take on the ellipse, the distance to the fixed points will always equal 10. As one distance gets longer the other will get shorter so that the sum will always be 10.

Equation of the horizontal ellipse of center (0, 0):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ where } b^2=a^2-c^2$$

$a>b$

This ellipse has the **major** axis is the x-axis making it open longer across. The length of the major axis is $2a$.
The **minor** axis is the y-axis and has length $2b$.
The **foci** points are $2c$ units apart.
The **center** of the ellipse is at (0, 0).
The **vertex** points are at the end points of the major axis.



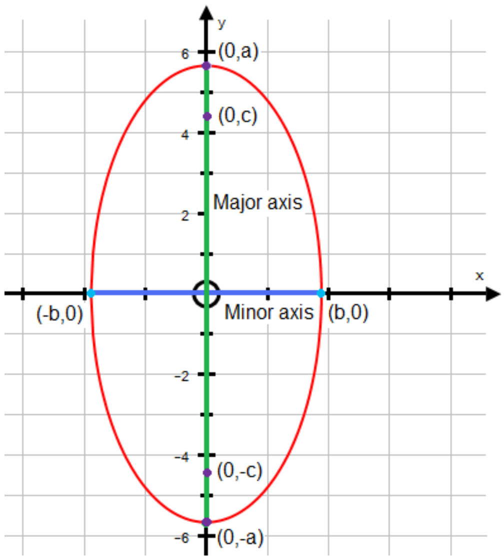
Look at the equation. The a value is always the **biggest** number!!

Equation of the vertical ellipse of center (0, 0):

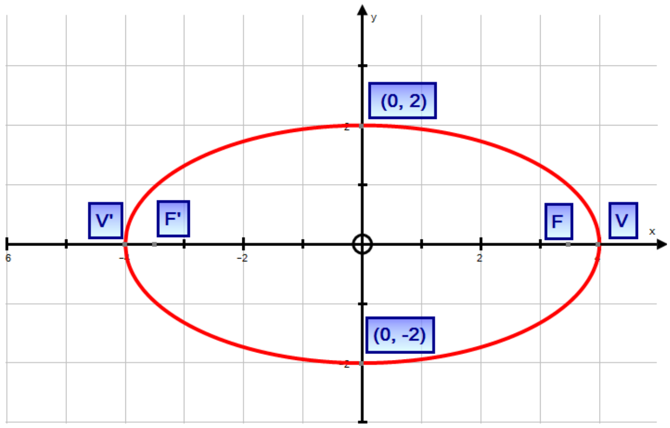
$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1 \text{ where } b^2=a^2-c^2$$

$a>b$

The major axis and the minor axes are reversed. The longer axis is now vertical.
What causes this to happen? Look at the equation closely. The a value is now under the y value rather than the x value in the previous equation.
We have an easy method to tell which way the ellipse opens; Look to see whether the larger value is under the x value or the y value!!



Example 1: Sketch the graph and find the vertices, end points of the minor axis and foci points for: $x^2 + 4y^2 = 16$



First put the equation in the correct form by dividing by16. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

The larger value is $a^2 = 16$ and $b^2 = 4$. Since the larger value is under x, the ellipse has the major axis **horizontal**. The values are a = 4, b = 2.

To find c, $c^2 = a^2 - b^2 = 16 - 4 = 12$

$\Rightarrow c = \sqrt{12} = 3.5$

Center at (0, 0)

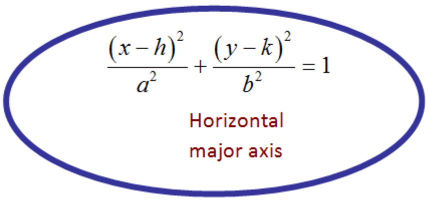
Vertices: (a,0) and (-a,0) ⇒(4, 0) and (-4, 0)

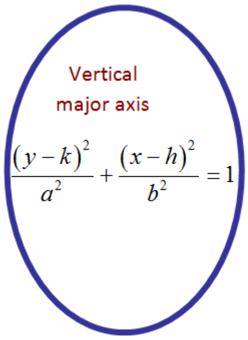
End points of minor axis: (0,b) and (0,-b) ⇒ (0, 2) and (0, -2)

Foci: (c,0) and (-c,0) ⇒ ($\sqrt{12}$, 0) and (-$\sqrt{12}$, 0)

Translation of the Ellipse

The center is now at (h, k). All values are now calculated from this point rather than from (0, 0)





$a > b$ and $c^2 = a^2 - b^2$

Elements of an ellipse: For each ellipse all the elements listed below should be identified:

- **Center**: Its coordinates are (h, k)
- **Vertices** located at the endpoints of the major axis
- **Foci** located between the center and the vertices
- **Major axis**: Length of major axis A'A = 2a
- **Minor Axis**: Length of minor axis B'B = 2b
- Distance from center C to focus F or F' is $c = \sqrt{a^2 - b^2}$
- **Eccentricity** = $\varepsilon = \frac{c}{a}$