Double Angle Formulas

The double-angle formulas can be quite useful when we need to simplify complicated trigonometric expressions.
With these formulas, it is better to remember where they come from, rather than trying to memorize the actual formulas. In this way, you will understand it better and have less to clutter your memory with
Recall from the last section, the sine of the sum of two angles:
$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
We will use this to obtain the sine of a double angle. If we take the left hand side (LHS):
$\sin(\alpha+\beta)$ and replace β with α, we get:
$\sin(\alpha+\beta)=\sin(\alpha+\alpha)=\sin 2\alpha$

Consider the RHS:
$\sin\alpha\cos\beta+\cos\alpha\sin\beta$. If we replace β with α, we obtain: $\sin\alpha\cos\alpha+\cos\alpha\sin\alpha=2\sin\alpha\cos\alpha$

Rule 1: Putting our results for the LHS and RHS together, we obtain the important result:
$\sin 2\alpha=2\sin\alpha\cos\alpha$

Using a similar process, we can start with the cosine of the sum of two angles
$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ we obtain the **cosine of a double angle** formula:

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$
By using the result $\sin^2 \alpha + \cos^2 \alpha = 1$, (which we found in Trigonometric Identities) we can write the RHS of the above formula as:
$\cos^2\alpha-\sin^2\alpha=(1-\sin^2\alpha)-\sin^2\alpha=1-2\sin^2\alpha$

Likewise, we can substitute $(1 - \cos^2 \alpha)$ for $\sin^2 \alpha$ into our RHS and obtain:
$\cos^2\alpha-\sin^2\alpha=\cos^2\alpha-(1-\cos^2\alpha)=2\cos^2\alpha-1$

Rule 2: The following have equivalent value, and we can use whichever one we like, depending on the situation:
$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$

$\cos 2\alpha=2\cos^2\alpha-1$

$\cos 2\alpha=1-2\sin^2\alpha$

The double –angle formulas are not restricted t angles 2θ and θ. Other double combinations. Such as 4θ and 2θor 6θ and 3θ, are also valid. Here are two examples.

$\text{Sin } 4\theta = 2\sin 2\theta \cos 2\theta$ and $\cos 6\theta = \cos^2 3\theta = \sin^2 3\theta$

By using double-angle formulas together with the sum formulas given in the preceding section, you can from other multiple—angle formulas.

Example 1: Write $\sin 3\alpha$ **in terms of** $\sin\alpha$ only

$\sin 3\alpha = \sin(2\alpha + \alpha)$
$= \sin 2\alpha \cos\alpha + \cos 2\alpha \sin\alpha$
$= (2\sin\alpha \cos\alpha)\cos\alpha + (1 - 2\sin^2\alpha)\sin\alpha$
$= 2\sin\alpha \cos^2\alpha + \sin\alpha - 2\sin^3\alpha$
$= 2\sin\alpha(1 - \sin^2\alpha) + \sin\alpha - 2\sin^3\alpha$
$= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha$
$= 3\sin\alpha - 4\sin^3\alpha$

Rule 3: $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$