Conditional Probability

In many situations, once more information becomes available; we are able to revise our estimates for the probability of further outcomes or events happening.

For example, suppose you go out for lunch at the same place and time every Friday and you are served lunch within 2O minutes with probability 0.85. However, given that you notice that the restaurant is exceptionally busy, the probability of being served lunch within 20 minutes may reduce to 0.6. This is the conditional probability of being served lunch within 20 minutes given that the restaurant is exceptionally busy.

Definition 1: The usual notation for "event A occurs given that event B has occurred" is A|B (A given B). The symbol | is a vertical line and does not imply division. P (A|B) denotes the probability that event A will occur given that event B has occurred already.

Rule 1: A rule that can be used to determine a conditional probability from unconditional probabilities is:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

where:

$P(A|B)$ = the (conditional) probability that event A will occur given that event B has occurred already

$P(A\cap B)$ = the (unconditional) probability that event A and event B occur

P(B) = the (unconditional) probability that event B occurs

Example 1: A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

$$\text{P(Second | First)}=\frac{\text{P(First and Second)}}{\text{P(First)}}=\frac{0.52}{0.42}=0.6$$

Rule 2: The conditional probability of an event B in relationship to an event A is the probability that event B occurs given that event A has already occurred. The notation for conditional probability is P (B|A), read as *the probability of B given A*. The formula for conditional probability is:

$$P(B|A)=\frac{P(A\cap B)}{P(A)}$$