Applications of Linear Systems

A wide variety of real world problems result in linear systems.
Before you can construct and solve equations that model real world applications, you need to be able to decide which quantities are most relevant to the problem.

More specifically, as you read through the original problem statement, it's advisable to make a list of the quantities that fall into the following categories:
- **Known values**: You can keep track of the known values by using number values that stay fixed throughout the problem
- **Unknown values**: You can keep track of the unknown values by using variables.

Example 1: A restaurant manager wants to purchase 200 sets of dishes. One design costs 25 Dhs per set, and another design costs 45 Dhs per set. Her total budget for buying dishes is 7400 Dhs. How many sets of each design should she purchase?

We have to construct a system of linear equations.

Step 1: Identify the desired variables.
The question is how many of each set of dishes can be purchased with the available budget.
So we can start by naming two variables
x = the number of sets of the 25 Dhs design
y = the number of sets of the 45 Dhs design

Step 2: Identify the known relationships in the problem.
 The problem statement tells us that the manager wants to buy a total of 200 sets of dishes.
 We can translate this fact as follows:

$\Rightarrow$ Total number of sets of dishes = 200 $\qquad \Rightarrow \qquad x + y = 200$
   The problem statement also tells us the price for each dish design.
   Using our variables named above, this means that the money spent
   on each dish design will be given by the equation
$\Rightarrow$ Total money spent = $25x + 45y$
   We also know that the manager has a total of 7400 Dhs to spend on the
   dishes. So we get the equation: $25x + 45y = 7400$

We now have a "2 by 2" linear system:
$$\begin{cases} x + y = 200 \\ 25x + 45y = 7400 \end{cases}$$
Let us solve the system by substitution

Using the first equation we find y in terms of x: $x + y = 200 \Rightarrow y = 200 - x$

Substituting in the second equation we find the value of x

$25x + 45y = 7400$

$25x + 45(200 - x) = 7400$

$25x + 9000 - 45x = 7400$

$-20x + 9000 = 7400$

$-20x = 7400 - 9000$

$20x = 1600$

$x = 80$

To obtain the value of y we substitute in the first equation:

$x + y = 200 \Rightarrow y = 200 - x = 200 - 80 = 120$

Therefore, the number of sets of the 25 Dhs design is 80 and the number of sets of the 45 Dhs design is 120.