Antiderivatives and Indefinite Integration

Finding an antiderivative is an 'inverse process' of finding a derivative. Instead of being given a function, and asking to find it's derivative, we're asking to find the function that our function is the derivative of. We have special notation and rules for finding antiderivatives.

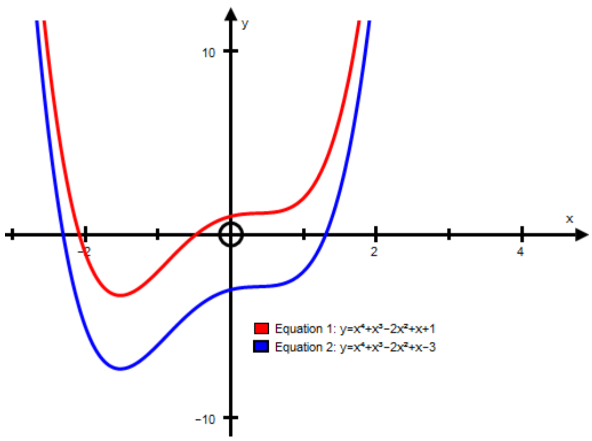
F(*x*) is the **antiderivative** of *f*(*x*) if $F'(x) = f(x)$.

Let's take a simple function like $f'(x) = 2x$. We know from experience that $f'(x) = 2x$ is the derivative of the function $f(x) = x^2$, therefore we call $f(x) = x^2$ the **antiderivative** of $f'(x) = 2x$

Example 1: Given: $g(x) = 4x^3 + 3x^2 - 4x + 1$, show that its antiderivative is $f(x) = x^4 + x^3 - 2x^2 + x + 2$
To show $f(x)$ is the antiderivative of $g(x)$ we need to take the derivative of $f(x) = x^4 + x^3 - 2x^2 + x + 2$.

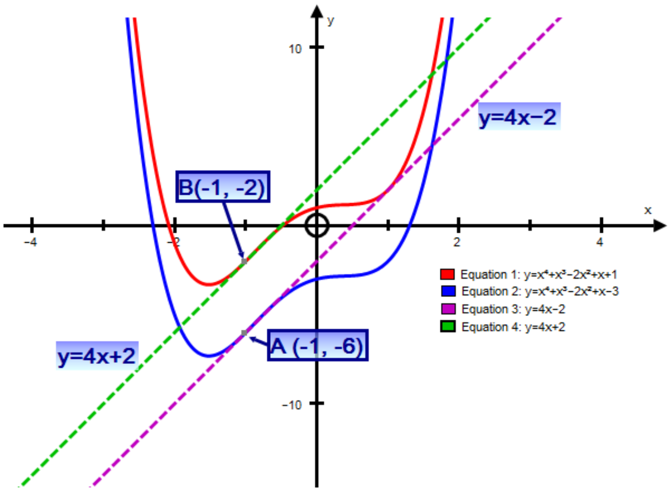
$$\frac{d}{dx} f(x) = \frac{d}{dx}\left(x^4 + x^3 - 2x^2 + x + 2\right) = 4x^3 + 3x^2 - 4x + 1 = g(x),$$

Since $\frac{d}{dx} f(x) = g(x)$, $f(x)$ is the antiderivative of $g(x)$.
Recall that the derivative of a function tells us about the slope of a line tangent to the graph of the function at some value *x* = *a*. The graphs of a few functions of the form $f(x) = x^4 + x^3 - 2x^2 + x + C$ are shown below.
Graphs of $f(x) = x^4 + x^3 - 2x^2 + x + 1$ and $g(x) = x^4 + x^3 - 2x^2 + x - 3$



Since the shape of each curve is the same and the curves are just translations of one another, we would predict that the slope of the tangent lines at all values of *x* = *a* must be the same for $f(x)$ and $g(x)$. The figure below has the tangent line drawn for both curves at *x* = −1.

Graphs of tangent lines to $f(x) = x^4 + x^3 - 2x^2 + x + 1$ and $g(x) = x^4 + x^3 - 2x^2 + x - 3$ at *x* = −1.



Since the two tangent lines at *x* = –*1* are parallel, $f'(-1) = g'(-1)$. Thus, we conclude that $f'(a) = g'(a)$ for all values of *x* = *a*.

We can also show this algebraically:

$f'(x) = 4x^3 + 3x^2 - 4x + 1$ and $g'(x) = 4x^3 + 3x^2 - 4x + 1$.

So, which function, $f(x)$ or $g(x)$ is the antiderivative of $4x^3 + 3x^2 - 4x + 1$? The answer is that both $f(x)$ and $g(x)$ are antiderivatives of $4x^3 + 3x^2 - 4x + 1$. As a matter of fact, any function of the form $f(x) = x^4 + x^3 - 2x^2 + x + C$, where C is some constant, is an antiderivative of $4x^3 + 3x^2 - 4x + 1$. Thus, the **general antiderivative** of $h(x) = 4x^3 + 3x^2 - 4x + 1$ is $H(x) = x^4 + x^3 - 2x^2 + x + C$.

This process of anti-differentiation is also known as **integration**. Integration uses the symbol $\int$ (called the integral sign) to denote that the antiderivative of a function is to be taken. When the integral sign is used we always find the most general antiderivative of the function that immediately follows the integral sign. The arbitrary constant *C* that comes from finding the most general antiderivative is called the **constant of integration**.

Notation: $\int f(x)dx$ is the **indefinite integral** of *f*(*x*) with respect to x. We refer to *f*(*x*) as the *integrand* in this context. 'Evaluating' an indefinite integral is finding it's antiderivative:

$$\int f(x)dx = F(x) + C .$$