Analyzing Graphs of Functions

The knowledge of some fairly simple graphs can help us graph some more complicated graphs. Many functions have graphs that are simple transformations of the common graphs.

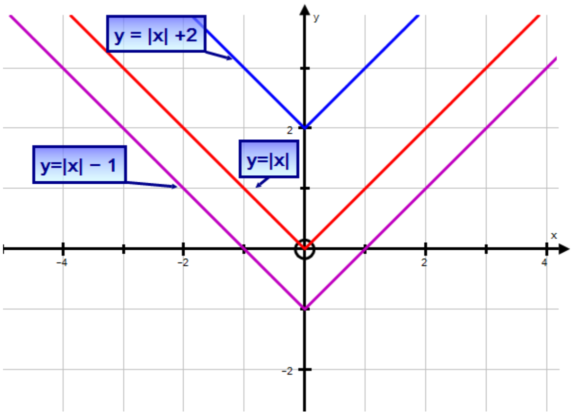
Part A: Shifting

Vertical Shifts: Let c be a positive real number. Vertical shifts in the graph of $y = f(x)$ are represented as follows

Vertical shift c units *upward*: $h(x) = f(x) + c$

Vertical shift c units *downward*: $h(x) = f(x) - c$

Example 1: Graph the functions $h(x) = |x|$, $g(x) = |x| - 1$, $f(x) = |x| + 2$



Horizontal Shifts: Let c be a positive real number. Horizontal shifts in the graph of $y = f(x)$ are represented as follows

Horizontal shift c units *left*: $h(x) = f(x + c)$

Horizontal shift c units *right*: $h(x) = f(x - c)$

Note: Now we can also combine the two shifts. If we know the graph of $f(x)$ the graph of $g(x) = f(x+c)+k$ will be the graph of $f(x)$ shifted left or right by *c* units depending on the sign of *c* and up or down by *k* units depending on the sign of *k*.

The graph of g(x) is shifted horizontally 3 units to the right and vertically 2 units upward from the graph of f(x).

Part B Reflecting

Reflection about the *x*-axis: Given the graph of $f(x)$ then the graph of $g(x) = -f(x)$ is the graph of $f(x)$ *reflected* about the *x*-axis. This means that the signs of all *y* coordinates are changed to the opposite sign.

Reflection about the *y*-axis: Given the graph of $f(x)$ then the graph of $g(x) = f(-x)$ is the graph of $f(x)$ *reflected* about the *y*-axis. This means that the signs of all *x* coordinates are changed to the opposite sign.

Part C Stretching

If a function $f(x)$ is given, then the graph of $kf(x)$ is a vertical stretch of the graph of $f(x)$ by a factor of k for $k > 1$, and a vertical compression for $0 < k < 1$.

What about $k < 0$?
If k<0, then we have stretching or compression and reflection about the x-axis.

Part D Symmetry

We've some fairly simply tests for each of the different types of symmetry.

1. A graph will have symmetry about the *x*-axis if we get an equivalent equation when all the $y's$ are replaced with $-y$.
2. A graph will have symmetry about the *y*-axis if we get an equivalent equation when all the $x's$ are replaced with $-x$.
3. A graph will have symmetry about the origin if we get an equivalent equation when all the $y's$ are replaced with $-y$ and all the $x's$ are replaced with $-x$

Definition 1: **Even Function:** A function is even if it is symmetric about the y-axis. Mathematically, since the y-coordinates (values of the function) have to be equal, and the x-coordinates are opposite, one can write: **f(-x) = f(x)**.

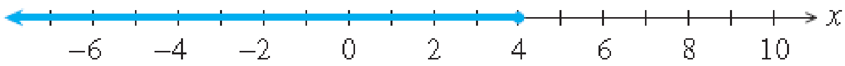
There is a reason these functions are called even. If you have a polynomial function in one variable, all of the exponents on the independent variable will be even. Remember that a constant is the zero power of the variable and zero is even.

Definition 2: **Odd Function:** A function is odd if it is symmetric about the origin. Mathematically, since both the x-coordinate and the y-coordinates are negated, one can write: **f(-x) = -f(x)**.

There is a reason why these functions are called odd. If you have a polynomial function in one variable, all of the exponents on the independent variable will be odd.

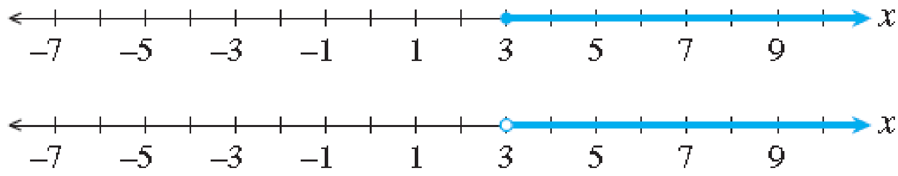
Graphing Piecewise Functions

On a number line, you can easily represent $x \le 4$, read "*x* is less than or equal to 4," by drawing a *closed dot* at 4 and shading all numbers less than (to the left of) 4 on the number line.



If the problem were changed to all numbers less than 4 ($x < 4$), you need to adjust your graph so that 4 is excluded. Your first instinct might be to draw a dot at 3 and shade to the left of it, but that would be incorrect because *x* could be one of the many numbers between 3 and 4 such as 3.2 or 3.9999. Aware of this, mathematicians have agreed to shade to the left of an *open dot* at 4 to represent $x < 4$.

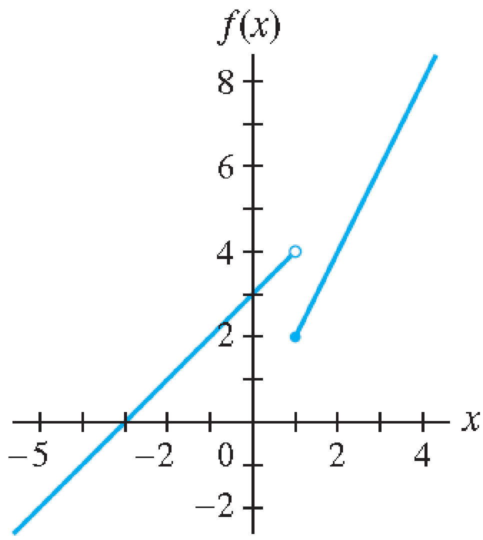
Similarly, a closed dot would be used for $x \geq 3$, read "x is greater than or equal to 3," and for $x > 3$, read "x is greater than but not equal to 3," shade to the right of an open dot at 3.



This system of dots and lines is easily transferred to the coordinate plane where we graph lines and other functions. One special kind of function is ***a piecewise continuous function***. An example is

$$f(x) = \begin{cases} x+3 & \text{if } x<1 \\ 2x & \text{if } x \geq 1 \end{cases}$$

These functions use inequalities to tell you when to use one "piece" of the function ($x + 3$ if $x < 1$) versus when to use the other "piece" ($2x$ if $x \geq 1$).When graphing a piecewise continuous function, take each "piece" and treat it as a separate function. We call the place on the graph where the function changes from one piece to the other the *break point*. When graphing a piecewise function with one break point, graph the left portion up to the break point, ending that portion with an open or closed dot depending on whether the < or ≤ symbol is used. Begin graphing the right piece of the function with an open or closed dot depending on whether the > or ≤ symbol is used. In the given function f the break point is at x=1. Notice that when $x = 1$ is substituted in $x + 3$, we obtain $f(x) = 4$. So an open dot is placed at the culmination of the left portion of the graph, the point (1, 4). When $x = 1$ is substituted in the bottom piece, $f(x) = 2$ results. Because the inequality for the right portion is ≥ , a closed dot must be placed at (1, 2). This point is the start of the right portion of the graph.



Definition 3:Linear piecewise functions: is the function defined by two or more different equations applied to different parts of the function's domain.

Notice that it appears to be composed of three segments, each a different linear function over a particular domain. Please note a filled circle includes that point, while an open circle does not include that point.