Adding & Subtracting Rational Expressions

Part A: Adding (or subtracting) rational expressions with the same denominators

Rational expressions with like denominators are added or subtracted in the same way as number fractions with like denominators. In the following example, two steps are used to subtract $\frac{2}{5}$ from $\frac{3}{5}$

Step 1 Add or subtract the numerators. $\frac{3}{5}-\frac{2}{5}=\frac{3-2}{5}$

Step 2 Write the sum or difference over the common denominator. $\frac{1}{5}$

You can use this same method to add or subtract rational expressions with like denominators. Recall that in rational expressions, both the numerator and denominator can have variables.

Rule 1: To add two or more rational expressions having the same denominators we follow the steps:
Step 1: Add all the numerators
Step 2: Write the common denominator once.
Step 3: Simplify the rational expression obtained if possible.

Example 1: Add or subtract:

1) $\frac{3x+1}{x+2}+\frac{x-2}{x+2}+\frac{5x-2}{x+2}$

$\frac{3x+1}{x+2}+\frac{x-2}{x+2}+\frac{5x-2}{x+2}$

$\frac{3x+1+x-2+5x-2}{x+2}$ Add all the numerators
Write the common denominator once

$\frac{9x-3}{x+2}$ The sum is in its simplest form

Part B: Adding (or subtracting) rational expressions with the different denominators

To add fractions with different denominators we find the LCM (least common multiple) of the denominators.

$$\frac{3}{7}+\frac{2}{5}=\left(\frac{3}{7}\right)\left(\frac{5}{5}\right)+\left(\frac{2}{5}\right)\left(\frac{7}{7}\right)=\frac{15}{35}+\frac{14}{35}=\frac{29}{35}$$

For rational expressions we find the LCD (least common denominator). The LCD is built up of all the factors of the individual denominators, each factor included the most number of times it appears in an individual denominator. The product of all the denominators is always a common denominator, but not necessarily the LCD (the final answer may have to be reduced).

The LCD of $\frac{-4}{3m}+\frac{2m}{m+1}$ is $3m(m+1)$

$$\frac{-4}{3m}+\frac{2m}{m+1} \qquad m\neq 0; m\neq -1$$

$$=\left(\frac{-4}{3m}\right)\left(\frac{m+1}{m+1}\right)+\left(\frac{2m}{m+1}\right)\left(\frac{3m}{3m}\right) \qquad \text{LCD}=3m(m+1)$$

$$=\frac{-4m-4}{3m(m+1)}+\frac{6m^2}{3m(m+1)}$$

$$=\frac{6m^2-4m-4}{3m(m+1)}$$

Rule 2: Use the following steps to add or subtract rational expressions with unlike denominators.

Step 1: Find the LCD.
Step 2: Change each rational expression into an equivalent expression using the LCD.
Step 3: Add or subtract as with rational expressions with like denominators.
Step 4: Simplify if necessary.