Verifying Trigonometric Identities

To prove Trigonometric Identities follow the steps listed below:

Step1: Learn well the basic trigonometric identities (or at least, know how to find them quickly). The better you know the basic identities, the easier it will be to recognize what is going on in the problems.

Step2: Work on the *most complex* side and simplify it so that it has the same form as the simplest side.

Step3: Don't **assume** the identity to **prove** the identity. This means don't work on both sides of the equals side and try to meet in the middle. Start on one side and make it look like the other side.

Step4: Many of these come out quite easily if you express everything on the most complex side in terms of **sine** and **cosine** only.

Step5: In most examples where you see power 2 (that is, ^{2}), it will involve using the identity $\sin^2 \theta + \cos^2 \theta = 1$

Using these suggestions, you can simplify and prove expressions involving trigonometric identities.

Example 1: Simplify: $\cot^2\theta - \csc^2\theta$

Since we have squaring then we have to think about the Pythagorean identities. $1+\cot^2\theta = \csc^2\theta$

$$\cot^2\theta - \csc^2\theta = \cot^2\theta - \left(1+\cot^2\theta\right) = \cot^2\theta - 1 - \cot^2\theta = -1$$

$$\cot^2\theta - \csc^2\theta = -1$$

Example 2: Simplify: $\dfrac{\sec^2\theta - 1}{\sin^2\theta}$

Since we have squaring then we have to think about the Pythagorean identities.
$1+\tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$

$$\frac{\sec^2\theta - 1}{\sin^2\theta} = \frac{\tan^2\theta}{\sin^2\theta} = \frac{\sin^2\theta / \cos^2\theta}{\sin^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

Example 3: Verify the identity: $\cos\theta + \sin\theta\tan\theta = \sec\theta$

$$\cos\theta + \sin\theta\tan\theta$$

$$= \cos\theta + \sin\theta\frac{\sin\theta}{\cos\theta}$$

$$= \cos\theta + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$