Trapezoids

Definition 1: A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. In a trapezoid the parallel sides are called **bases** and the nonparallel sides are called **legs**. Each trapezoid has two pairs of **base angles**. In trapezoid *TRAP*, $\angle T$ and $\angle R$ are one pair of base angles; $\angle P$ and $\angle A$ are the other pair.

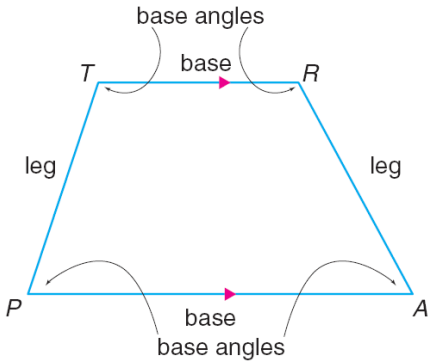
$\overline{TR} \parallel \overline{PA}$

$\overline{TR}$ and $\overline{PA}$ are the two bases

$\overline{TP}$ and $\overline{PA}$ are the legs

$\angle$T and $\angle$R is one pair of base angles

$\angle$P and $\angle$A is another pair of base angles

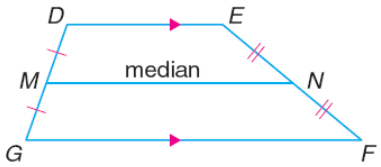


The angles whose vertices are the vertices of the longer base are called the lower base angles, and the other two angles are called the other two angles are called the upper base angles.

Definition 2: The **median** of a trapezoid is the segment that joins the midpoints of its legs.
In the figure, $\overline{MN}$ is the median.

M is the midpoint of $\overline{DG}$

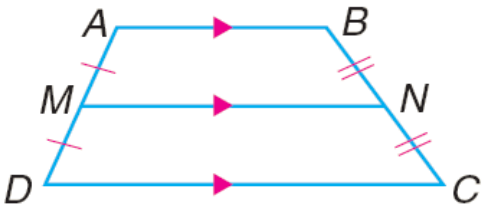
N is the midpoint of $\overline{EF}$



Theorem 1: The median of a trapezoid is parallel to the bases, and the length of the median equals one-half the sum of the lengths of the bases.

$\overline{AB} \parallel \overline{MN}, \overline{DC} \parallel \overline{MN}$

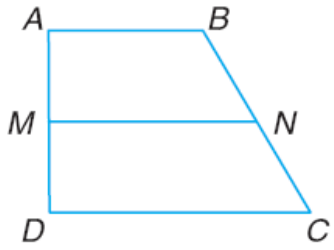
$MN = \frac{1}{2}(AB + DC)$



Example 1: Find the length of median MN in trapezoid ABCD if AB = 15 and DC = 25.

MN is the median in trapezoid ABCD

$$\Rightarrow MN = \frac{1}{2}(AB + DC)$$
$$\Rightarrow MN = \frac{1}{2}(15 + 25)$$
$$\Rightarrow MN = \frac{1}{2}(40)$$
$$\Rightarrow MN = 20$$

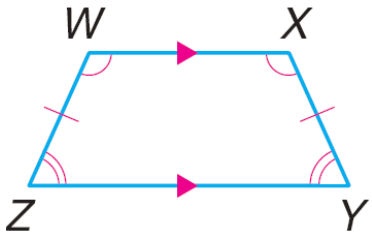


In a triangle if two sides are equal then the triangle is called an isosceles triangle, and as we know from before if a triangle is isosceles then the base angles are congruent. Similarly if the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.

Definition 3: A trapezoid with two non-parallel sides congruent is called an **isosceles trapezoid**.

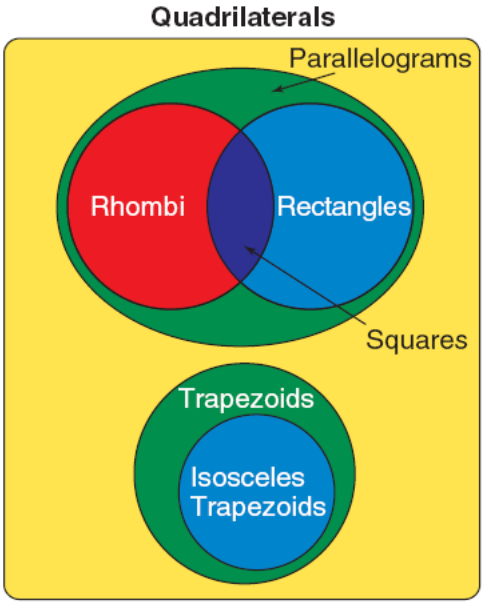
Theorem 2: Each pair of base angles in an isosceles trapezoid is congruent.

$$\angle W \cong \angle X, \angle Z \cong \angle Y$$



Theorem 3: The diagonals of an isosceles trapezoid are congruent.

We have studied quadrilaterals, parallelograms, rectangles, rhombi, squares, trapezoids, and isosceles trapezoids. The Venn diagram illustrates how these figures are related.



- The Venn diagram represents all quadrilaterals.

- Parallelograms and trapezoids do not share any characteristics except that they are both quadrilaterals. This is shown by the non-overlapping regions in the Venn diagram.

- Every isosceles trapezoid is a trapezoid. In the Venn diagram, this is shown by the set of isosceles trapezoids contained in the set of trapezoids.

- All rectangles and rhombi are parallelograms. Since a square is both a rectangle and a rhombus, it is shown by overlapping regions.