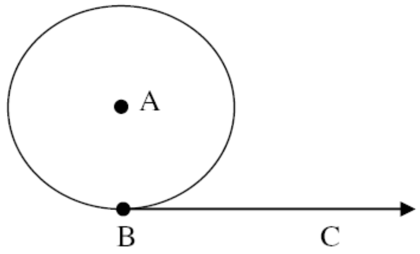
Tangents and Secants

Definition 1: In a plane, a line is tangent to the circle if and only if it intersects a circle in exactly one point.

BC is **tangent** to□ *A*, because the line containing *BC* intersects the circle in exactly one point.

Point B is called the **point of tangency**.



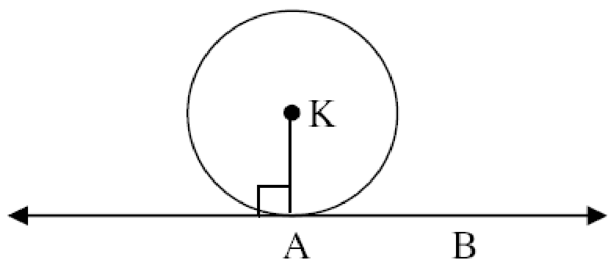
Definition 2: A secant of a circle is a line that intersects a circle in two points.

Postulate: At a given point on a given circle, one and only one line can be drawn that is tangent to the circle.

Tangent Theorem: In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

If *AB* is a tangent□ K at A

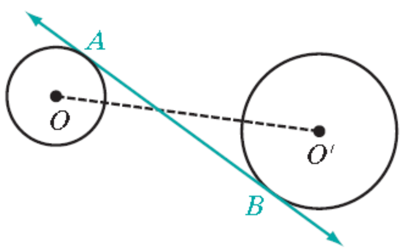
$\Rightarrow AB \perp AK$



Converse of Tangent Theorem: In a plane, if a line is perpendicular to a radius of a circle at the endpoint on the circle, then the line is tangent to the circle.

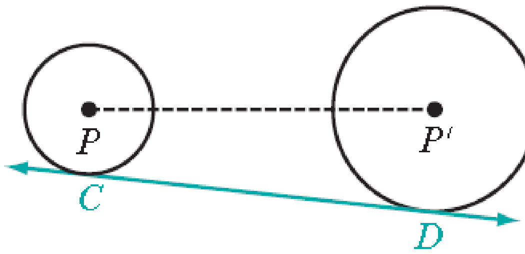
Common Tangents

Definition 3: A common tangent is a line that is tangent to each of two circles.

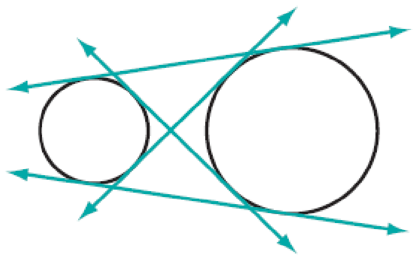
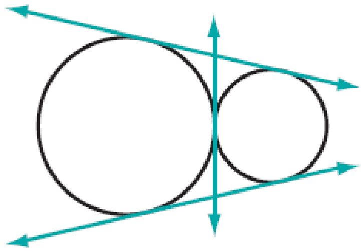
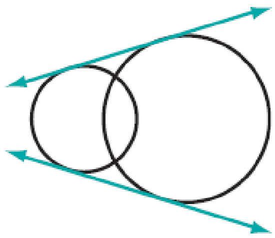
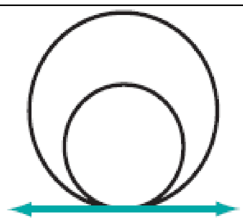
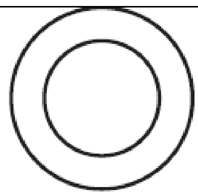


In the diagram, $\overleftrightarrow{AB}$ is tangent to the circle of center O at A and to the circle of center O' at B. Tangent $\overleftrightarrow{AB}$ is said to be a **common internal tangent** because the tangent intersects the line segment joining the centers of the circles.

In the diagram, $\overleftrightarrow{CD}$ is tangent to the circle of center P at C and to circle of center P' at D. Tangent $\overleftrightarrow{CD}$ is said to be a **common external tangent** because the tangent does not intersect the line segment joining the centers of the circles.



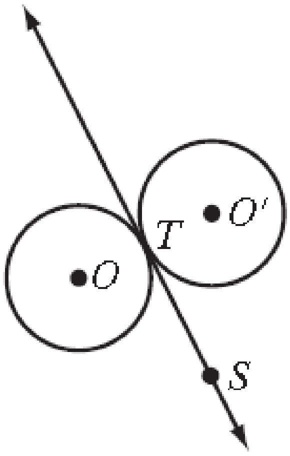
The diagrams below show that two circles can have four, three, two, one or no common tangents.

The two circles have 4 common tangents **Two internal tangents** **Two external tangents**	
The two circles have 3 common tangents **One internal tangent** **Two external tangents**	
The two circles have 2 common tangents **Two external tangents**	
The two circles have one common tangent **One tangent only**	
The two circles have no common tangents	

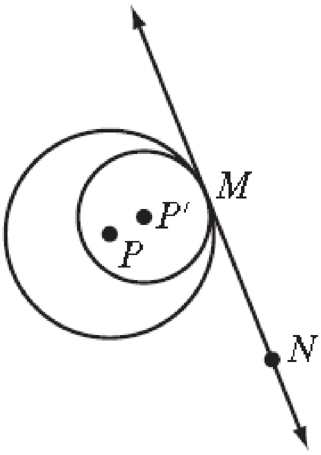
Tangent Circles:

Definition 4: Two circles are tangent to each other if they are tangent to the same line at the same point.

In the diagram, $\overleftrightarrow{ST}$ is tangent to the circle of center O and to the circle of center O' at T. The two circles are **tangent externally** because every point of one of the circles, except the point of tangency, is an external point of the other circle.

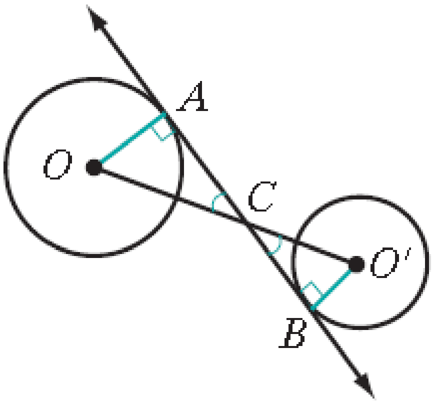


In the diagram, $\overleftrightarrow{MN}$ is tangent to the circle of center P and to the circle of center P' at M. The two circles are **tangent internally** because every point of one of the circles, except the point of tangency, is an internal point of the other circle.



Example 1:

Given: $C(O,OA)$ and $C(O',O'B)$



$\overleftrightarrow{AB}$ is the common tangent to C at A and C′ at B

$\overline{OO'}$ intersects $\overleftrightarrow{AB}$ at point C

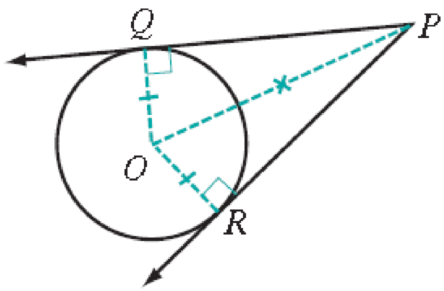
Prove: $\frac{AC}{BC}=\frac{OC}{O'C}$

Proof:

Statements	Reasons
1) $\overleftrightarrow{AB}$ is the common tangent to C at A	1) Given
2) $\overleftrightarrow{AB} \perp \overline{OA}$	2) A tangent is $\perp$ to the radius at point of tangency
3) $\angle OAC$ is a right angle	3) Definition of perpendicular lines
4) $\overleftrightarrow{AB}$ is the common tangent to C′ at B	4) Given
5) $\overleftrightarrow{AB} \perp \overline{O'B}$	5) A tangent is $\perp$ to the radius at point of tangency
6) $\angle O'BC$ is a right angle	6) Definition of perpendicular lines
In $\triangle OCA$ and $\triangle O'CB$, we have:	
7) $\angle OAC \cong \angle O'BC$	7) All right angles are congruent
8) $\angle OCA \cong \angle O'CB$	8) Vertically opposite angles
9) $\triangle OCA \sim \triangle O'CB$	9) AA postulate
10) $\frac{AC}{BC} = \frac{OC}{O'C}$	10) The length of corresponding sides of similar $\triangle$s are proportional

Tangent from an Exterior Point Theorem: Tangent segments drawn to a circle from an external point are congruent.

Example 2:



Given: $\overleftrightarrow{PQ}$ is tangent to circle with center O at Q.

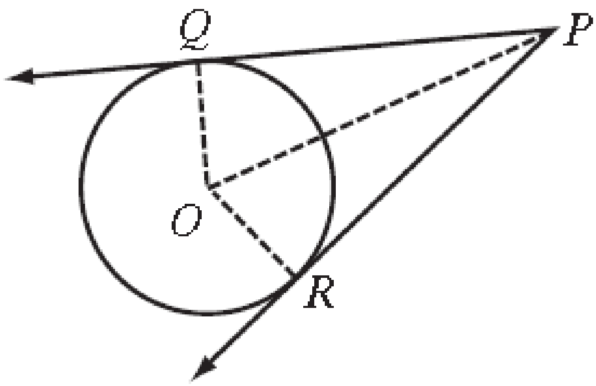
$\overleftrightarrow{PR}$ is tangent to circle with center O at R.

Prove: $\overline{PQ} \cong \overline{PR}$

Proof:

Statements	Reasons
In $\triangle OPQ$ and $\triangle OPR$, we have:	
1) $\overline{OQ} \cong \overline{OR}$	1) Radii of the same circle
2) $\overleftrightarrow{PQ}$ is tangent to circle with center O at Q.	2) Given
3) $\overleftrightarrow{PQ} \perp \overline{OQ}$	3) A tangent is $\perp$ to the radius at point of tangency
4) $\angle OQP$ is a right angle	4) Definition of perpendicular lines
5) $\overleftrightarrow{PR}$ is tangent to circle with center O at R.	5) Given
6) $\overleftrightarrow{PR} \perp \overline{OR}$	6) A tangent is $\perp$ to the radius at point of tangency
7) $\angle ORP$ is a right angle	7) Definition of perpendicular lines
8) $\triangle OPQ \cong \triangle OPR$	8) HL Theorem
9) $\overline{PQ} \cong \overline{PR}$	9) CPCTC

Corollary 1: If two tangents are drawn to a circle from an external point, then the line segment from the center of the circle to the external point bisects the angle formed by the tangents.

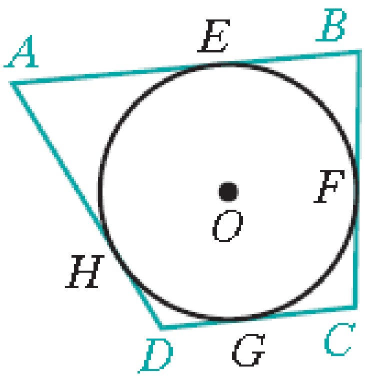


Corollary 2: If two tangents are drawn to a circle from an external point, then the line segment from the center of the circle to the external point bisects the angle whose vertex is the center of the circle and whose rays are the two radii drawn to the points of tangency.

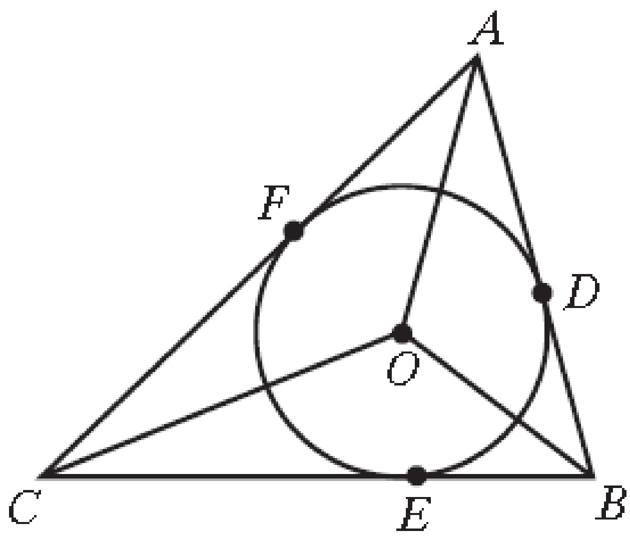
Definition 5: A polygon is **circumscribed** about a circle if each side of the polygon is tangent to the circle.

When a polygon is **circumscribed** about a circle, we also say that the circle is **inscribed** in the polygon.

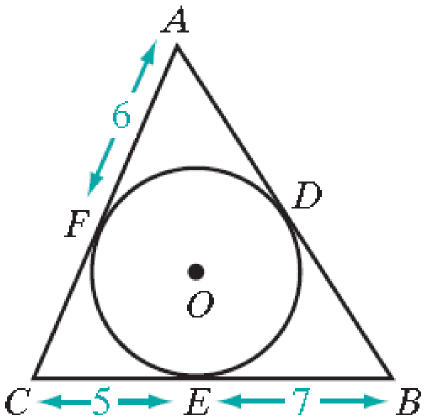
In the diagram below, $\overline{AB}$ is tangent to the circle of center O at E, $\overline{BC}$ is tangent to the circle of center O at F, $\overline{DC}$ is tangent to the circle of center O at G, and $\overline{AD}$ is tangent to the circle of center O at H. Therefore, ABCD is **circumscribed** about the circle of center O. Moreover, the circle of center O is the **inscribed** in quadrilateral ABCD.



If $\triangle ABC$ is **circumscribed** about the circle of center O, then we know that $\overline{OA}$, $\overline{OB}$,and $\overline{OC}$ are the bisectors of the angles of $\triangle ABC$, and O is the point at which the angle bisectors of the angles of a triangle intersect.



Example 3: $\overline{AB}$, $\overline{BC}$,and $\overline{CA}$ are tangent to the circle of center O at D, E, and F respectively. If AF=6 cm, BE=7 cm and CE=5 cm, find the perimeter of $\triangle ABC$



Tangent segments drawn to a circle from an external point are congruent.

$AD = AF = 6$, $BD = BE = 7$, $CF = CE = 5$

Therefore, $AB = AD + DB = 6 + 7 = 13$; $BC = BE + EC = 7 + 5 = 12$; $CA = CF + FA = 5 + 6 = 11$.

$Perimeter\ of\ \triangle ABC = AB + BC + CA = 13 + 12 + 11 = 36$