# Spheres

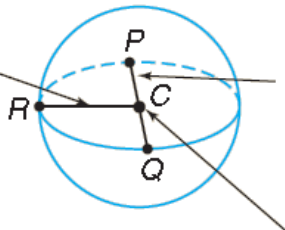
A circle is the set of all points in a plane that are a fixed distance from a point in the plane, called the center. Suppose you were not limited to a plane. From a center, all points in space at a fixed distance form a hollow shell called a **sphere**. The hollow shell of the sphere is its surface.

Definition 1: A sphere is the set of all points that are a fixed distance from a given point called the center.

A sphere with center C:


A sphere has many properties like those of a circle.

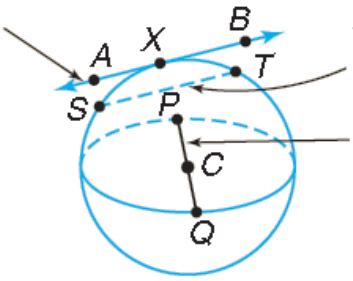
A *radius* of a sphere is a segment whose endpoints are the center and a point on the sphere. $\overline{CR}$ is a radius.




A *diameter* of a sphere is a segment that joins two points on the sphere and passes through its center. $\overline{PQ}$ is a diameter.

Point *C* is the *center* of the sphere.

A *tangent* to a sphere is a line that intersects the sphere at exactly one point. $\overleftrightarrow{AB}$ is tangent to the sphere at point *X*.




A *chord* of a sphere is a segment whose endpoints are points on the sphere. $\overline{ST}$ and $\overline{PQ}$ are chords.

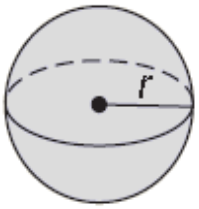
The diameter $\overline{PQ}$ is a special type of chord.

**Definition 2:** **Surface Area of a Sphere:** If a sphere has a surface area of *S* square units and a radius of *r* units, then $S = 4\pi r^2$.

**Definition 3:** **Volume of a Sphere:** If a sphere has a volume of *V* cubic units and a radius of *r* units, then $V = \frac{4}{3}\pi r^3$

$$S = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$




**Example 1:** The radius of a sphere is 6.7 cm. Find the volume.

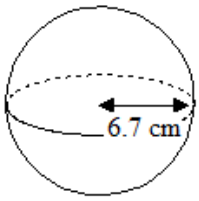
Substitute *r* = 6.7 cm into the formula

Volume = $\frac{4}{3}\pi r^3$

V = $\frac{4}{3} \times \pi \times 6.7^3$

V = 1259.833

V = 1260 cm$^3$




**Example 2:** Find the volume of the hemisphere given its diameter = 18.4 cm

The diameter of the hemisphere is 18.4 cm.
Therefore the radius is 9.2 cm.

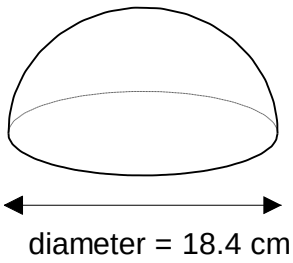
Volume of the hemisphere = $\frac{1}{2} \times$ volume of sphere

= $\frac{1}{2} \times \frac{4}{3}\pi r^3$

= $\frac{1}{2} \times \frac{4}{3} \times \pi \times 9.2^3$

= $\frac{1}{2} \times 3261.76$

= 1630 cm$^3$ (to 3 SF)



**Example 3:** A sphere has a volume of 86.5 cm$^3$. Find the radius of the sphere.

Substitute into the formula for the volume of a sphere: Volume = $\frac{4}{3}\pi r^3$

$86.5 = \frac{4}{3}\pi r^3$

So, $86.5 = 4.18879 r^3$

i.e., $20.65035 = r^3$

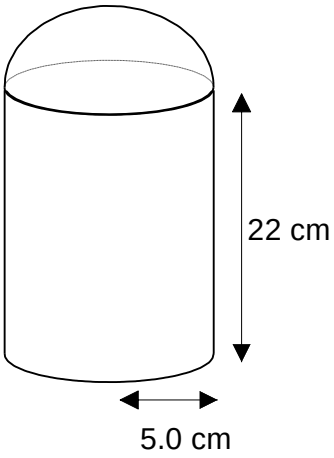
So, $r = 2.74$ cm

The sphere has radius 2.74 cm.

**Example 4:** The object shown is made up from a cylinder and a hemisphere.

The cylinder has radius 5.0 cm and height 22 cm.

Find the volume of the object.




Volume of cylinder = $\pi r^2 h$

$= \pi \times 5^2 \times 22$

= 1728 cm$^3$

The hemisphere must also have radius 5 cm.

Volume of the hemisphere = $\frac{1}{2} \times$ volume of sphere

$= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$

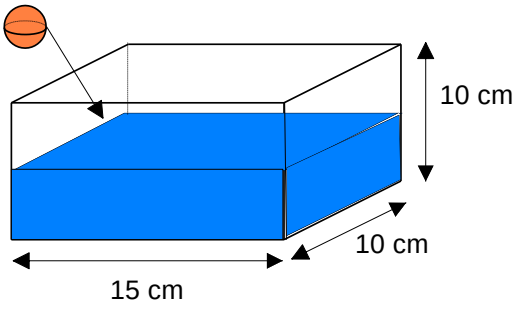
= 262 cm$^3$

Therefore total volume of the object = 1728 + 262 = 1990 cm$^3$.

**Example 5:** A tank measures 15 cm by 10 cm by 10 cm

The tank is half-full of water.

A solid metal sphere with radius 2 cm is placed into the tank.



Assuming that the sphere sinks to the bottom of the tank, calculate the amount by which the water level in the tank rises

As the sphere will be completely submerged, it will displace its volume of water.

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times 2^3$ = 33.51 cm$^3$.

Therefore the water displaced is 33.51 cm$^3$.

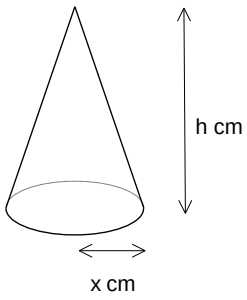
The water displaced has the form of a cuboid with measurements 15 cm by 10 cm by *h* cm, where *h* is the height by which the water level rises.

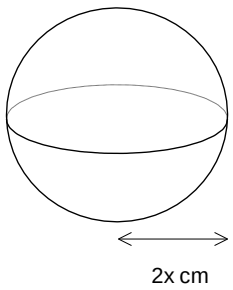
So $15 \times 10 \times h = 33.51$

i.e. $h = 0.22$ cm

The water rises by 0.22 cm.

**Example 6:** The radius of the base of a cone is *x* cm and its height is *h* cm.
The radius of a sphere is 2*x* cm.




Diagrams NOT accurately drawn

The volume of the cone and the volume of the sphere are equal.
Express *h* in terms of *x*.
Give your answer in its simplest form.

The volume of the cone is $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi x^2 h$

The volume of the sphere is $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2x)^3$ (note: the brackets around 2*x* are important)

$= \frac{4}{3}\pi\times 8x^3$ (cubing both 2 and *x*)

$= \frac{32}{3}\pi x^3$

As the sphere and the cone have the same volume, we can form an equation:

$\frac{1}{3}\pi x^2 h = \frac{32}{3}\pi x^3$

$\pi x^2 h = 32\pi x^3$ (multiplying both sides by 3)

$x^2 h = 32x^3$ (dividing both sides by π)

$h = 32x$ (diving both sides by $x^2$)