Solving Rational Equations

Every week there is a 10-point quiz in the math class. For the first five quizzes, Hala scored a total of 36 points, which gave her an average of 7.2. She is determined to get 10 points on each of the next quizzes until she brings her average up to an 8. On how many quizzes must she score 10 points in order to have an overall quiz average of 8 points?

Let x be the number of quizzes on which she must score 10 points.
Total number of quizzes = $5 + x$
Sum of scores = $36 + 10x$

$$Average = \frac{36+10x}{5+x}$$

$$8 = \frac{36+10x}{5+x}$$

The equation above is a rational equation because it contains at least one rational expression.

$$8(5+x) = 36+10x$$

$$40+8x = 36+10x$$

$$40-36 = 10x-8x$$

$$4 = 2x$$

$$x = 2$$

Rule 1: There are three steps to solve rational equations.

Step 1: Find the LCD of each term.
Step 2: Multiply each side of the equation by the LCD.
Step 3: Use the Distributive Property to simplify.

Example 1: Solve each equation:

1) $\frac{3a}{5}+\frac{3}{2}=\frac{7a}{10}$

$$\Rightarrow \frac{6a+15}{10}=\frac{7a}{10}$$

$$\Rightarrow 6a+15=7a$$

$$\Rightarrow 6a-7a=-15$$

$$\Rightarrow -a=-15 \Rightarrow a=15$$

2) $\frac{2}{3x}-\frac{1}{2x}=\frac{1}{6}$

$\frac{2}{3x}-\frac{1}{2x}=\frac{1}{6}$ $\quad x\neq 0$

$\frac{4-3}{6x}=\frac{1}{6}$

$\frac{1}{6x}=\frac{1}{6}$

$6x=6$

$x=1$

3) $\frac{3}{r}-\frac{1}{r-1}=\frac{1}{r-1}$

$\frac{3}{r}-\frac{1}{r-1}=\frac{1}{r-1}$ $\quad r\neq 0,\ \ r\neq 1$

$\frac{3(r-1)-1(r)}{r(r-1)}=\frac{1}{r-1}$

$\Rightarrow \frac{3r-3-r}{r(r-1)}=\frac{1}{r-1}$

$\Rightarrow \frac{2r-3}{r(r-1)}=\frac{1}{r-1}$

$\Rightarrow (2r-3)(r-1)=r(r-1)$

$\Rightarrow 2r^2-2r-3r+3=r^2-r$

$\Rightarrow 2r^2-2r-3r+3-r^2+r=0$

$\Rightarrow r^2-4r+3=0$

$\Rightarrow (r-3)(r-1)=0$

$\Rightarrow r=3 \ \& \ r=1$

$r=1$ is rejected

$\Rightarrow r=3$ is the only solution