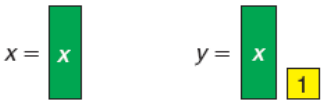
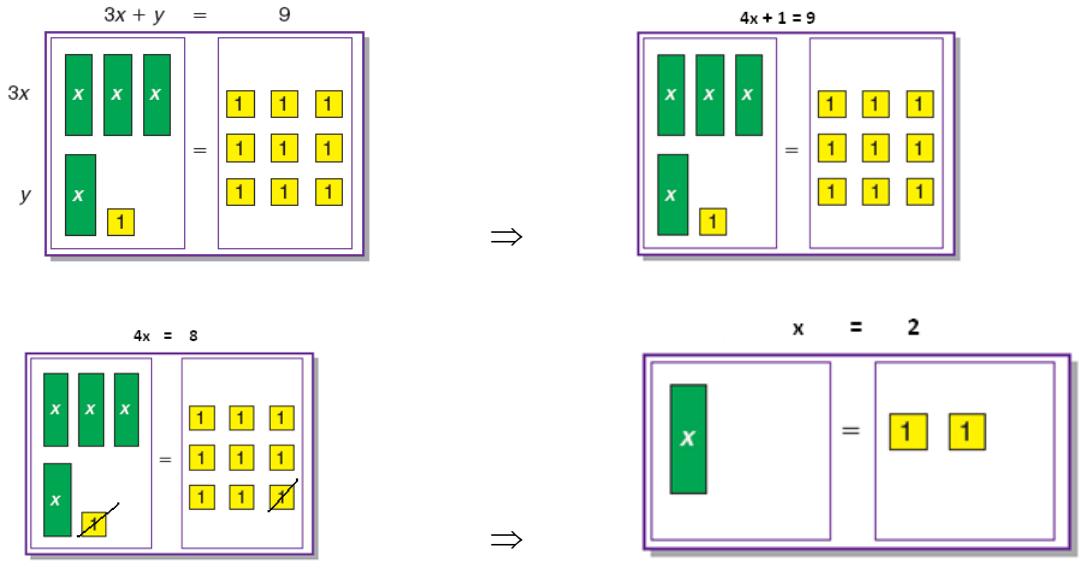
Solving Linear Systems by Substitution

Activity 1 : Use a model to solve the system $y = x + 1$ and $3x + y = 9$.

Step 1: Let a green tile represent x. Then, since $y = x + 1$, 1 green tile and 1 yellow tile represent y.



Step 2: Represent $3x + y = 9$ on the equation mat. On one side of the mat, place three green tiles for $3x$ and 1 green tile and 1 yellow tile for y. On the other side, place 9 yellow tiles.



Rule 1: The solution of a system can be found by using an algebraic method called the Substitution Method. To find the set of solutions using the substitution method, we follow the steps listed below:

Step 1: Simplify if needed (removing ( ) and removing fractions)
Step 2: Solve one equation for either variable.
Step 3: Substitute what you get for step 2 into the other equation.
Step 4: Solve for the remaining variable.
Step 5: Solve for second variable.
Step 6: Check the proposed ordered pair solution in BOTH original equations.

Remark:

1) If your variable drops out and you have a **FALSE** statement that means your answer is **no solution**.
2) If your variable drops out and you have a **TRUE** statement that means your answer is **infinite solutions**, which would be the equation of the line.

Example 1: Solve the system of equations by the substitution method.

$$\begin{cases} 4x-6y=12 \\ y=3x-1 \end{cases}$$

Both equations are in their simplified forms, we need to choice one equation to start with, we will use the second equation $y=3x-1$ and substitute the value of y in the first equation

$4x-6y=12$ subtitute $y=3x-1$

$4x-6(3x-1)=12$

$4x-18x+6=12$

$-14x+6=12$

$-14x=6$

$x=\frac{6}{-14}=-\frac{3}{7}$

We substitute the value of x in the second equation to get the value of y.

$y=3x-1$

$\Rightarrow y=3\left(-\frac{3}{7}\right)-1$

$\Rightarrow y=-\frac{9}{7}-1$

$\Rightarrow y=\frac{-9-7}{7}=-\frac{16}{7}$

Let us check if the solution $\left(-\frac{3}{7},-\frac{16}{7}\right)$ is acceptable.

$4x-6y=4\left(-\frac{3}{7}\right)-6\left(-\frac{16}{7}\right)=-\frac{12}{7}+\frac{96}{7}=\frac{-12+96}{7}=\frac{84}{7}=12$ True

$3x-1=3\left(-\frac{3}{7}\right)-1=-\frac{9}{7}-1=-\frac{16}{7}=y$ True

The solution of this system of equations is $\left(-\frac{3}{7},-\frac{16}{7}\right)$

Example 2: Solve the system of equations by the substitution method.

$$\begin{cases} y=6x-1 \\ y-6x=3 \end{cases}$$

Both equations are in their simplified forms, we need to choice one equation to start with; we will use the first equation $y=6x-1$ and substitute the value of y in the second equation

$y-6x=3$ substitute $y=6x-1$

$\Rightarrow 6x-1-6x=3$

$\Rightarrow -1=3$ false statement

The variable drops out and you have a FALSE statement, then there is no solution. If we were to graph these two, they would be parallel to each other.
There are no ordered pairs to check.
The answer is no solution.