Solutions of Systems of Equations

Graphs of systems of linear equations may be intersecting lines, parallel lines, or the same line. Systems of equations can be described by the number of solutions they have.

In general, a solution of a system in two variables is an ordered pair that makes BOTH equations true.

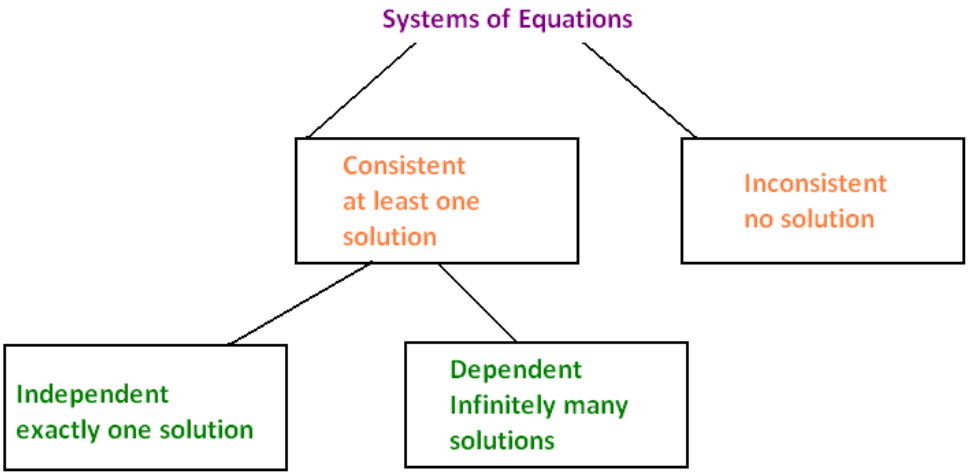
In other words, it is where the two graphs intersect, what they have in common. So if an ordered pair is a solution to one equation, but not the other, then it is NOT a solution to the system.

A **consistent** system is a system that has at least one solution.

An **inconsistent** system is a system that has no solution.

The equations of a system are **dependent** if ALL the solutions of one equation are also solutions of the other equation. In other words, they end up being the same line.

The equations of a system are **independent** if they do not share ALL solutions. They can have one point in common, just not all of them.



There are three possible outcomes that you may encounter when working with these systems:

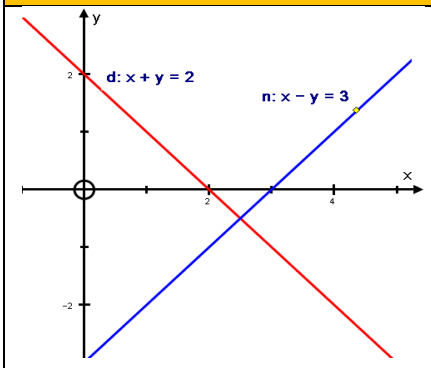
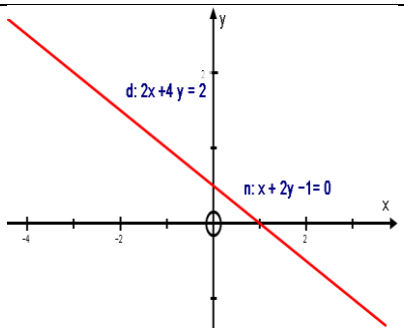
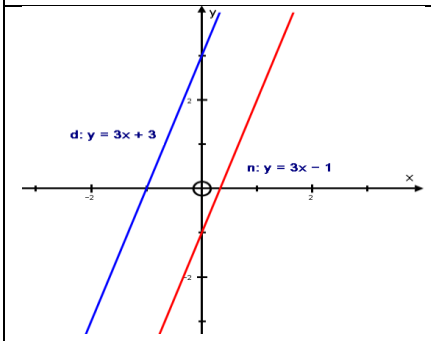
1) one solution
2) no solution
3) infinite solutions

1) One Solution
   If the system in two variables has one solution; it is an ordered pair that is a solution to BOTH equations. In other words, when you plug in the values of the ordered pair it makes BOTH equations TRUE.
   This is the result of a consistent system of independent equations

2) No Solution
If the two lines are parallel to each other, they will never intersect. This means they do not have any points in common. In this situation, you would have no solution.
This is the result of an inconsistent system of independent equations

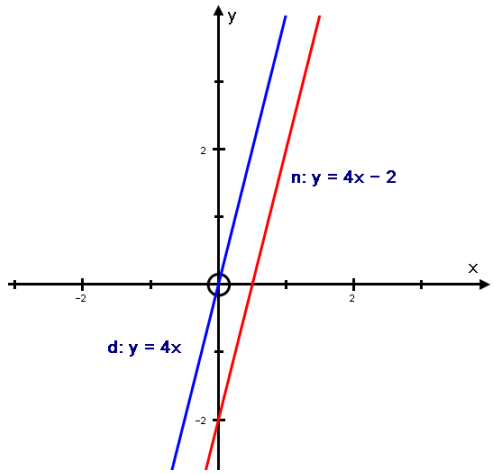
3) Infinite Solutions
If the two lines end up lying on top of each other, then there is an infinite number of solutions. In this situation, they would end up being the same line, so any solution that would work in one equation is going to work in the other.
This is the result of a consistent system of dependent equations

The different possibilities for the graphs of two linear equations are summarized in the following table:

Graph	Description of Graph	Slopes and Intercepts of Lines	Number of Solutions	Type of System
	**Intersecting Lines**	**Different Slopes**	**1**	**Consistent and Independent**
	**Same Line**	**Same Slope, Same Intercepts**	**Infinitely Many**	**Consistent and Dependent**
	**Parallel Lines**	**Same Slope, Different Intercepts**	**0**	**Inconsistent**

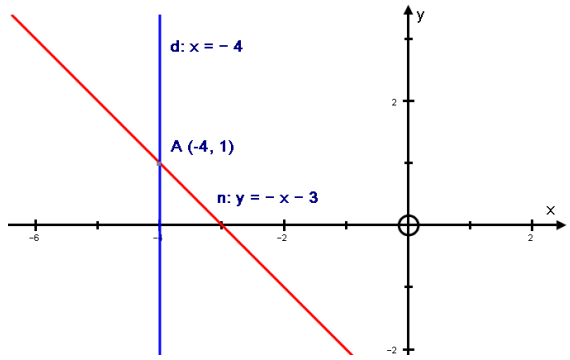
Example 1: State whether each system is consistent and independent, consistent and dependent, or inconsistent.

1)



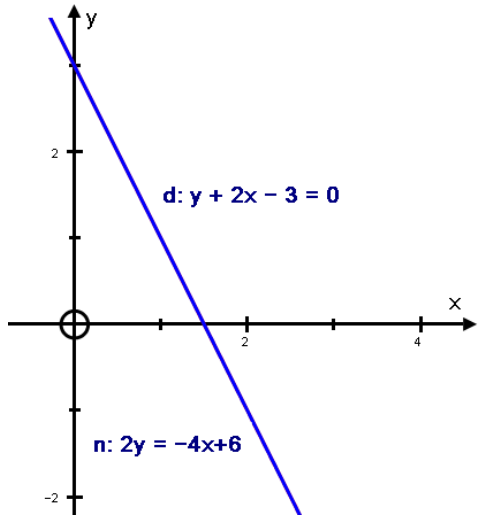
The graphs appear to be parallel lines. Since they do not intersect, there is no solution. This system is inconsistent.

2)



The graphs appear to intersect at the point at (-4, 1). Because there is one solution, this system of equations is consistent and independent.

3)



Both equations have the same graph.

Because any ordered pair on the graph will satisfy both equations, there are infinitely many solutions. The system is consistent and dependent.