

Similarity in Right Triangles

Definition 1: Geometric means: the geometric means x between two numbers a and b is the number which satisfies the proportion: $a : x = x : b$ or $\frac{a}{x} = \frac{x}{b}$ or $x^2 = ab$ or $x = \sqrt{ab}$

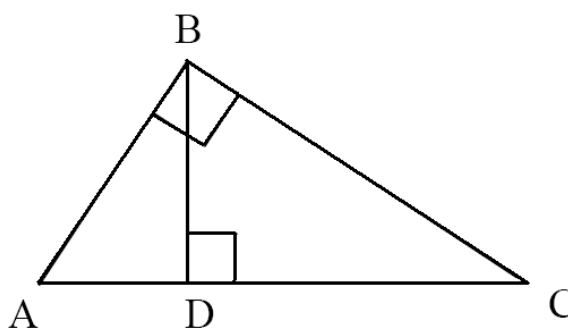
Example 1: Find the geometric mean between 7 and 11

$$\frac{7}{x} = \frac{x}{11} \quad \text{or} \quad x^2 = 77 \quad \text{or} \quad x = \sqrt{77}$$

Theorem 1: If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

\overline{BD} is an altitude relative to \overline{AC}

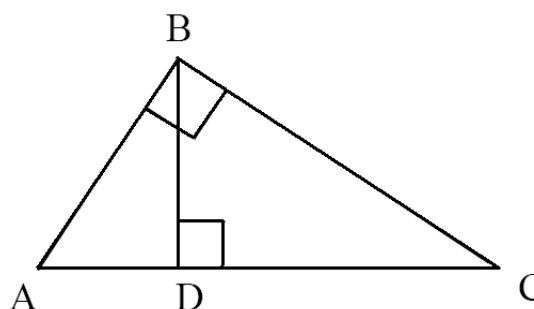
$$\Rightarrow \triangle ABC \sim \triangle ADB \sim \triangle BDC$$



Corollary 1: If the altitude is drawn to the hypotenuse of a right triangle, then the length of the altitude is the geometric means between the segments of the hypotenuse.

$$\frac{\text{part of hypotenuse}}{\text{altitude}} = \frac{\text{altitude}}{\text{part of hypotenuse}}$$

$$\frac{AD}{BD} = \frac{BD}{DC}$$



Corollary 2: If the altitude is drawn to the hypotenuse of a right triangle, then each leg is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to the leg.

$$\frac{\text{whole hypotenuse}}{\text{leg}} = \frac{\text{leg}}{\text{part of hypotenuse touching leg}}$$

$$\frac{AC}{AB} = \frac{AB}{AD}$$

$$\frac{AC}{BC} = \frac{BC}{DC}$$

