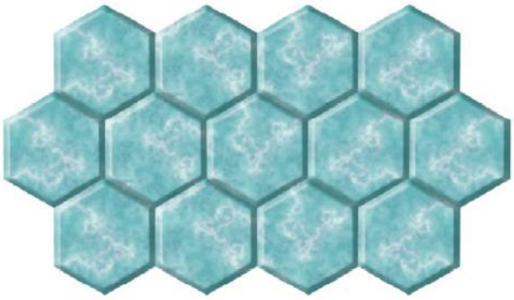
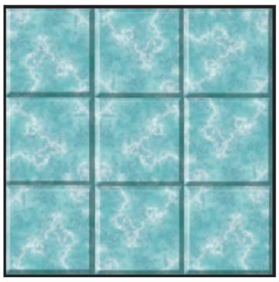
Similar Polygons

In *Shapes and Designs*, you learned that some polygons can fit together to cover, or tile, a flat surface. For example, the surface of a honeycomb is covered with a pattern of regular hexagons. Many bathroom and kitchen floors are covered with a pattern of square tiles.





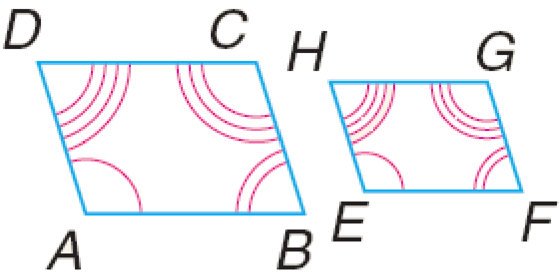
If we look at the second figure we notice that final shape is a square similar to the small pieces but of different size. Polygons that are the same shape but not necessarily the same size are called similar polygons.

Definition 1: Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

$$ABCD \sim EFGH$$
$$\Rightarrow \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

AND

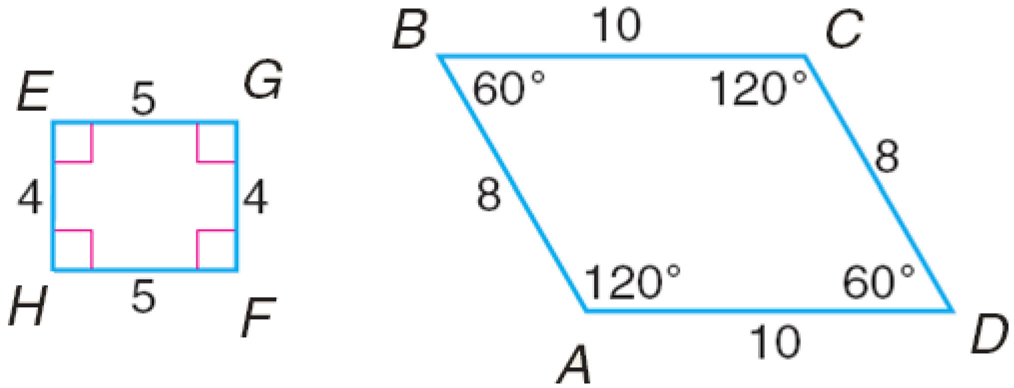
$$\angle A \cong \angle E, \angle B \cong \angle F, \angle C \cong \angle G, \angle D \cong \angle H$$



Remark 1: In congruent figures, corresponding angles and sides are congruent. In similar figures, corresponding angles are congruent, and the measures of the corresponding sides have equivalent ratios, or are proportional.

Rule 1: To determine if polygons are **similar**, you must do two things:
(1) Verify that corresponding angles are congruent
(2) Verify that corresponding sides are proportional.

Example 1: Determine if the polygons are similar. Justify your answer.



$$\frac{AB}{HE}=\frac{8}{4}=2\ ;\frac{BC}{EG}=\frac{10}{5}=2\ ;\frac{CD}{GF}=\frac{8}{4}=2\ ;\frac{DA}{FH}=\frac{10}{5}=2$$

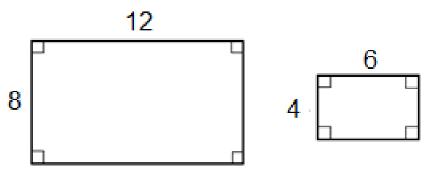
$$\Rightarrow\frac{AB}{HE}=\frac{BC}{EG}=\frac{CD}{GF}=\frac{DA}{FH}$$

The measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. Therefore, the polygons are not similar.

Scale drawings are often used to represent something that is too large or too small to be drawn at actual size. Contractors use scale drawings called *blueprints* to represent the floor plan of a house to be constructed. The blueprint and the floor plan are similar.

Definition 2: **Scale factor:** The ratio of the lengths of two corresponding sides of similar polygons.

$$\text{Scalar factor} = \frac{\text{Second corresponding side}}{\text{first corresponding side}} = \frac{6}{12} = \frac{1}{2}$$



Now, if the rectangles are truly similar, then it doesn't matter which corresponding sides we use to determine the scale factor. Let's test this by using the left sides of the rectangles.

$$\text{Scalar factor} = \frac{\text{Second corresponding side}}{\text{first corresponding side}} = \frac{4}{8} = \frac{1}{2}$$

The fact that we get the same scale factor means that they are the same proportions, which agrees with them being similar.

Remark 2: If the rectangles are given to you in a different order, you'd get a scale factor of 2. This shows how scale factors can be inverted, so you have to keep track of which you consider the first and second polygon.

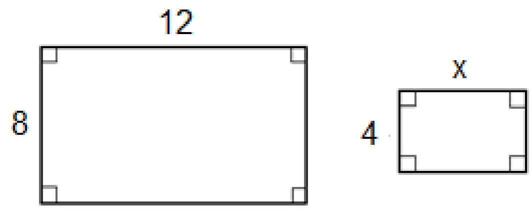
Example 2: The two rectangles are similar, find the value of x.

$$\frac{top \text{ side } 2}{top \text{ side } 1} = \frac{left \text{ side } 2}{left \text{ side } 1}$$

$$\Rightarrow \frac{x}{12} = \frac{4}{8}$$

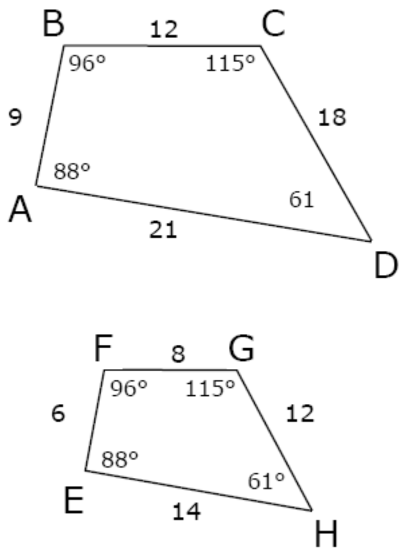
$$\Rightarrow 8x = 4(12)$$

$$\Rightarrow x = 6$$



Example 3: Are the two quadrilaterals similar? If so, state the similarity, and give the scale factor.

$$\frac{9}{6} = \frac{18}{12} = \frac{12}{8} = \frac{21}{14} = \frac{3}{2}$$



$$m\angle B = m\angle F = 96^0 \Rightarrow \angle B \cong \angle F$$

$$m\angle C = m\angle G = 115^0 \Rightarrow \angle C \cong \angle G$$

$$m\angle D = m\angle H = 61^0 \Rightarrow \angle D \cong \angle H$$

$$m\angle E = m\angle A = 88^0 \Rightarrow \angle E \cong \angle A$$

$\Rightarrow$quad ABCD ~ quad EFGH