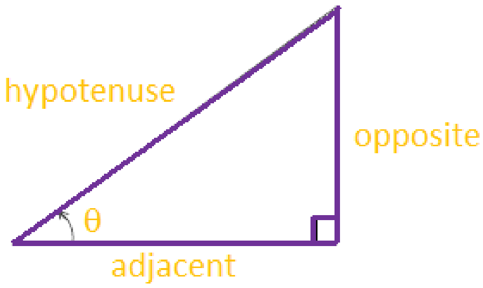
Right Triangle Trigonometry

Our second look at the trigonometric functions is from a right triangle perspective. Consider a right triangle, with one acute angle labeled θ . Relative to the angle θ , the three sides of the triangle are the hypotenuse, the opposite to side (the side opposite the angle θ),and the adjacent side(the side adjacent to the angle θ).



Using the lengths of these three sides, you can form **six ratios** that define the six trigonometric functions of the acute angle θ .

sine cosecant cosine secant tangent cotangent

In the following definitions, it is important to see that $0^0 < \theta < 90^0$ (θ lies in the first quadrant) and that for such angles the value of each trigonometric function is **positive.**

Definition: **Right Triangle Definition of Trigonometric Functions**: Let θ be an acute angle of a right triangle. The six trigonometric functions of the angle θ are defined as ratios of the sides of the right triangle:

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

$$\csc\theta = \frac{hyp}{opp} \qquad \sec\theta = \frac{hyp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

Note: The abbreviation opp., adj., and hyp. Represent the lengths of the three sides of a right triangle.

Opp =the length of the side **opposite** to θ
Adj = the length of the side **adjacent** to θ
Hyp = the length of the **hypotenuse**

To remember these, many people use SOH CAH TOA, that is:

$$\text{S}\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{C}\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \qquad \text{T}\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Example 1: Refer to the diagram to evaluate:

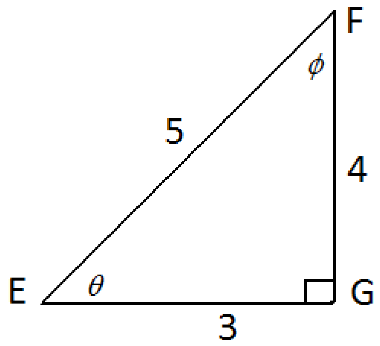
1) $\sin\theta = \frac{opp}{hyp} = \frac{GF}{EF} = \frac{4}{5}$

2) $\sin\phi = \frac{opp}{hyp} = \frac{EG}{EF} = \frac{3}{5}$

3) $\cos\theta = \frac{adj}{hyp} = \frac{EG}{EF} = \frac{4}{5}$

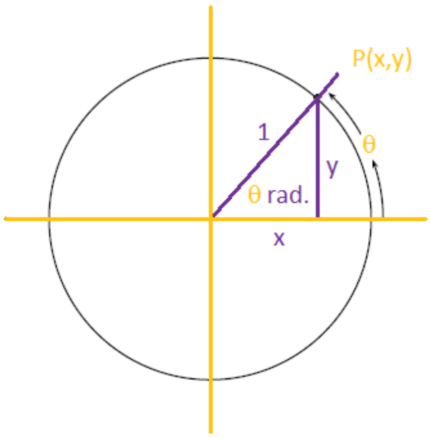
4) $\csc\theta = \frac{hyp}{opp} = \frac{EF}{GF} = \frac{5}{4}$

5) $\sec\phi = \frac{hyp}{adj} = \frac{EF}{GF} = \frac{5}{4}$



Relationship between trigonometry of right triangles and the unit circle.

Now, in a unit circle draw a right triangle



Using the right triangle definitions of the trigonometric functions, we can write:

$$\sin\theta = \frac{opposite}{hypotenuse} = \text{y} \quad \text{and} \qquad \cos\theta = \frac{adjacent}{hypotenuse} = \text{x}$$

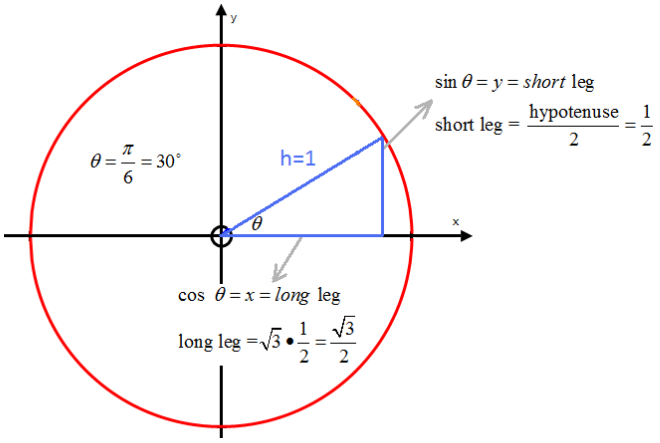
In other words, we see that $\cos\theta = x$ and $\sin\theta = y$ which is the definition of the trigonometric functions using the unit circle.

Once $\sin\theta$ and $\cos\theta$ are determined, the other trigonometric functions are defined in the same manner as in the right triangle approach.

From Geometry ... remember the ratios for 30-60-90 and 45-45-90 triangles?

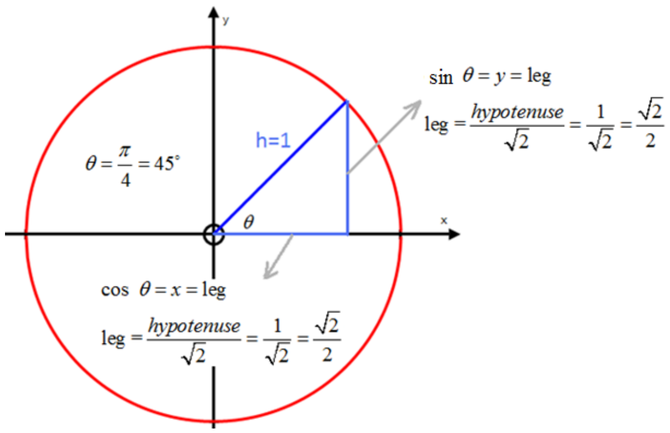
Now let's put these triangles onto a unit circle, and we'll be able to evaluate the trig functions at angles that are 30°, 60° and 45°. Remember, on a unit circle, the hypotenuse, r = 1, $\sin\theta = y$ and $\cos\theta = x$

Sine and Cosine of 30°



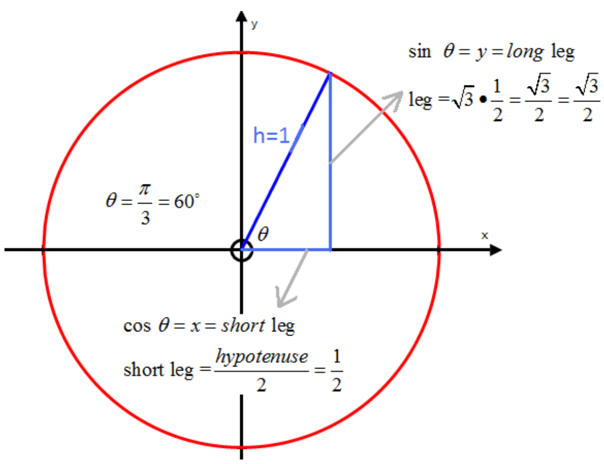
$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Sine and Cosine of 45°



$$\sin 45^\circ = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \frac{\sqrt{2}}{2}$$

Sine and Cosine of 60°



$$\sin 30^\circ = \frac{\sqrt{3}}{2} \qquad \cos 30^\circ = \frac{1}{2}$$

Example 2: A historical light house is 200 cm from a bike path along the edge of a lake. A walkway to the lighthouse is 400 cm long. Find the acute angle θ between the bike path and the walkway.

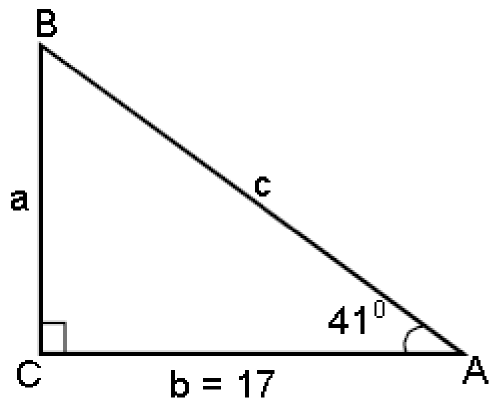
The sin of the angle θ is: $\sin\theta = \frac{opp.}{hyp.} = \frac{200}{400} = \frac{1}{2}$

Therefore, the value of the angle θ is 30°

Using the sine and cosine values we can find the values of the other four trigonometric functions. The table below will summarize all the values of the remarkable acute angles:

Angle (θ)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	1	0	$undef.$	1	$undef.$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	$undef.$	1	$undef.$	0

Example 3: Solve the right triangle shown in the figure



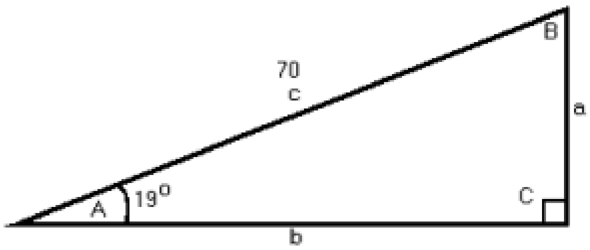
To find the value of c, we will use the cosine function which relates the adjacent side to the hypotenuse

$$\cos 41 = \frac{adj}{hyp} = \frac{b}{c} \Rightarrow c = \frac{b}{\cos 41} = \frac{17}{0.75} = 22.6$$

To find the value of a, we will use the tangent function which relates the adjacent side to the opposite

$\tan 41 = \frac{opp}{adj} = \frac{a}{b} \Rightarrow a = b\tan 41 = 17(0.87) = 14.79$

Example 4: Find length *b*.



b is adjacent to *angle* A.

By definition, $\cos\theta = \frac{adj}{hyp}$

Therefore, we write an equation using the cosine.

$\cos 19 = \frac{b}{70}$

Multiply each side by 70 to isolate the variable *b*.

$b = 70\cos 19 = 66.19$