Pyramid and Cone

Part A: Pyramid

Theorem 1: **Lateral Area of a Regular Pyramid:** If a regular pyramid has a lateral area of *L* square units, a base with a perimeter of *P* units, and a slant height of l units, then $L = \frac{1}{2}Pl$

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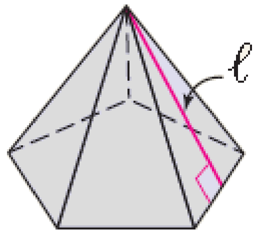

With prisms and cylinders, the formula for the surface area is $S = L + 2B$. Since a pyramid has only one base, the formula for its surface area is $S = L + B$, where $L = \frac{1}{2}Pl$.

Theorem 2: **Surface Area of a Regular Pyramid:** If a regular pyramid has a surface area of *S* square units, a slant height of l units, and a base with perimeter of *P* units and area of *B* square units, then $S = \frac{1}{2}Pl + B$

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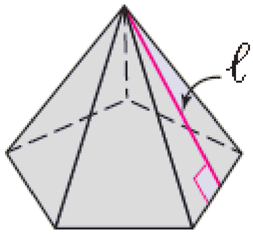

Theorem 3: **Volume of a Pyramid:** If a pyramid has a volume of *V* cubic units and a height of *h* units and the area of the base is *B* square units, then $V = \frac{1}{3}Bh$

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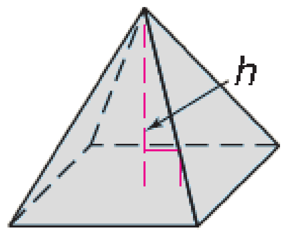
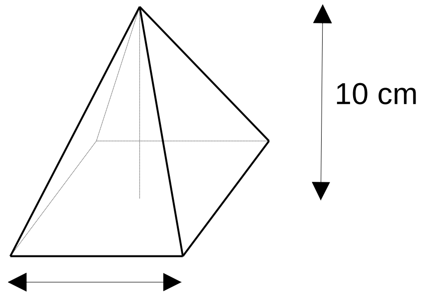


The relationship between the volumes of a cone and a cylinder is similar to the relationship between the volumes of a pyramid and a prism. The volume of a cone is $\frac{1}{3}$ the volume of a cylinder with the same base and height. The volume of a cylinder is $V = \pi r^2 h$, so the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

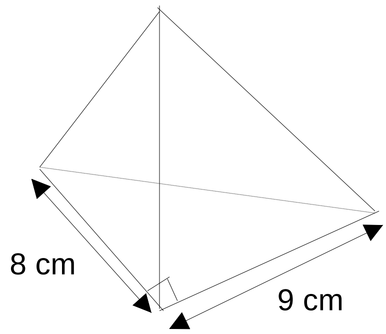
Example 1: The pyramid shown has a square base.
The square has sides of length 12 cm.
The height of the pyramid is 10 cm.
Find the volume.



The area of the square base is 12 × 12 = 144 cm^2
So, the volume of the pyramid is:

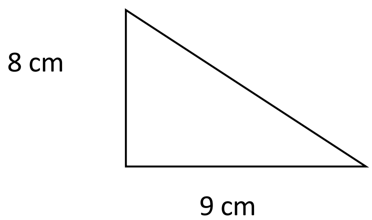
$$\text{Volume} = \frac{1}{3} \times 144 \times 10$$
$$= 48 \times 10$$
$$= 480 \text{ cm}^3.$$

Example 2: The diagram shows a triangular-based pyramid.



The base of the pyramid is a right-angled triangle.
The volume of the pyramid is 325 cm^3.
Find the height of the pyramid.

The base of the pyramid is as shown:



The area of the base is $\frac{1}{2} \times 9 \times 8 = 36$ cm^2.

Substitute information into the formula for the volume of a pyramid.

Volume of pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$

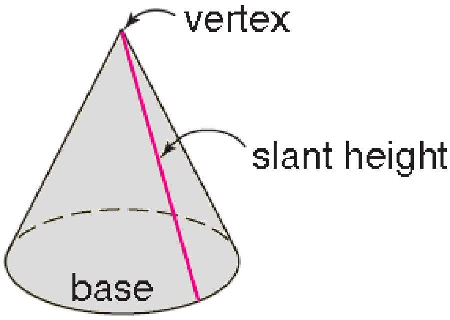
325 = $\frac{1}{3} \times 36 \times \text{height}$

325 = 12 × height.
So, height = 325 ÷ 12 = 27.08 cm

Part B: Cone

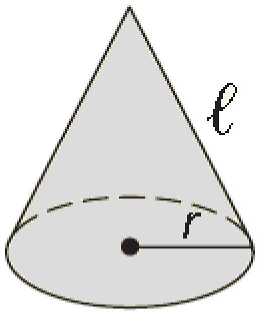
The slant height of a cone is the length of any segment whose endpoints are the vertex of the cone and a point on the circle that forms the base.

The formulas for finding the lateral area and surface area of a cone are similar to those for a regular pyramid. However, since the base is a circle, the perimeter becomes the circumference, and the area of the base is πr^2 square units.



Theorem 4: **Lateral Area of a Cone** If a cone has a lateral area of *L* square units, a slant height of l units, and a base with a radius of *r* units, then $L = \frac{1}{2} \bullet 2\pi rl$ or $L = \pi rl$

$L = \pi rl$



To find the surface area of a cone, add its lateral area and the area of its base.

Theorem 5: **Surface Area of a Cone** If a cone has a surface area of *S* square units, a slant height of l units, and a base with a radius of *r* units, then $S = \pi rl + \pi r^2$

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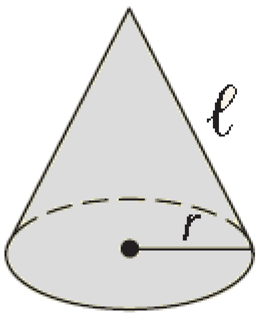

Theorem 6: **Volume of a Cone:** If a cone has a volume of *V* cubic units, a radius of *r* units, and a height of *h* units, then $V = \frac{1}{3}\pi r^2 h$

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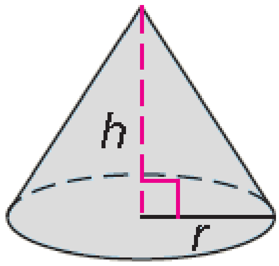
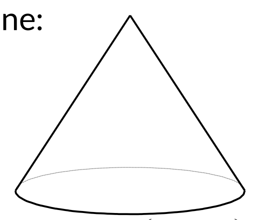


Example 3: The base of a cone has a radius of 4 cm. The height of the cone is 6 cm. Find the volume of the cone. Leave your answer in terms of π.

Substitute the information into the formula for the volume of a cone:



Volume of cone = $\frac{1}{3}\pi r^2 h$

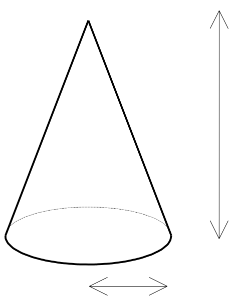
$= \frac{1}{3} \times \pi \times 4^2 \times 6$

$= 2 \times \pi \times 16$

Volume = 32π cm^3.

Example 4: A cone has a volume of 1650 cm^3. The cone has a height of 28 cm. Find the radius of the cone. Give your answer correct to 2 significant figures.

Substitute information into the formula:



Volume of cone = $\frac{1}{3}\pi r^2 h$

1650 = $\frac{1}{3} \times \pi \times r^2 \times 28$

1650 = $29.32153r^2$

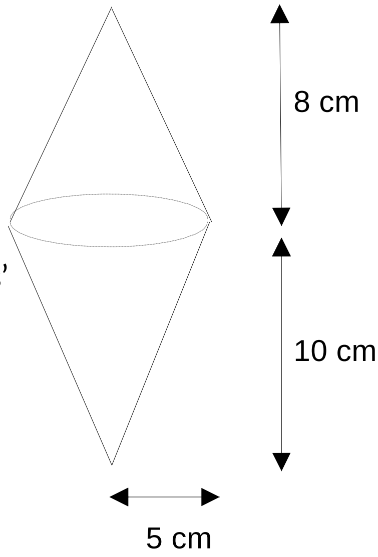
$r^2 = 56.2726$

i.e. r = 7.5 cm (to 2 SF)

The radius of the cone is therefore 7.5 cm.

Example 5: The diagram shows an object made from two cones, one on top of the other.

The top cone has a height of 8 cm and the bottom cone has a height of 10 cm. Both cones have a radius of 5 cm.



Find the total surface area of the object.

The formula for the curved surface area of a cone is: $\pi r l$.

We can find the slant length, l, for each cone using Pythagoras' theorem – we know the radius and the height of each cone.

Top cone:

$$l^2 = 5^2 + 8^2 = 25 + 64 = 89$$

$$l = \sqrt{89} = 9.434cm$$

Therefore, curved surface area = $\pi \times 5 \times 9.434 = 148.2cm$

Bottom cone:

$$l^2 = 5^2 + 10^2 = 25 + 100 = 125$$

$$l = \sqrt{125} = 11.180cm$$

Therefore, curved surface area = $\pi \times 5 \times 11.180 = 175.6cm$

So total surface area is $324cm^2$ (to 3SF)