Properties of Sets

Properties of Sets: The set operations verify the following properties:

Law 1: Associative Laws:

$A\cup(B\cup C)=(A\cup B)\cup C$

$A\cap(B\cap C)=(A\cap B)\cap C$

Law 2: Commutative Laws:

$A\cup B=B\cup A$

$A\cap B=B\cap A$

Example 1: Prove $A\cap B=B\cap A$

$A\cap B=\{x:x\in A \text{ and } x\in B\}=\{x:x\in B \text{ and } x\in A\}=B\cap A$

Law 3: Distributive Laws:

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

$A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Law 4: Identity Laws:

$A\cup\varnothing=A$

$A\cap U=A$

Law 5: Complement Laws:

$A\cup\overline{A}=U$

$A\cap\overline{A}=\varnothing$

Law 6: Idempotent Laws:

$A\cap A=A$

$A\cup A=A$

Law 7: Bound Laws:

$A\cup U=U$

$A\cap\varnothing=\varnothing$

Law 8: Absorption Laws:

$A\cup(A\cap B)=A$

$A\cap(A\cup B)=A$

Law 9: Involution Law:

$\overline{\overline{A}}=A$

Law 10: $0/1$ laws:

$\overline{\varnothing}=U$

$\overline{U}=\varnothing$

Law 11: DeMorgan's Laws:

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Even though the proofs of set equations are simple, they can be quite tedious, and we shall introduce a new tool for tackling problems of this sort.

If A and B are sets, then for an arbitrary element x of our universe of discourse U there are four possibilities:

x ∈ A and x ∈ B, x ∈ A and x ∉ B , x∉ A and x ∈ B, x ∉ A and x ∉ B.

For every one of these cases let us consider if x is in the intersection of A and B: If x ∈ A and x ∈ B, then x ∈ A∩B. In all three other cases, we have x ∉ A∩B.

This observation can be put in the form of a table which looks like this:

A	B	$A \cap B$	$A \cup B$
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

Example 2: Using the table method, prove:

The second law of absorption $A \cap (A \cup B) = A$

A	B	$A \cup B$	$A \cap (A \cup B)$
1	1	1	1
1	0	1	1
0	1	1	0
0	0	0	0

Example 3: Using the table method, prove:

The first distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ is:

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0