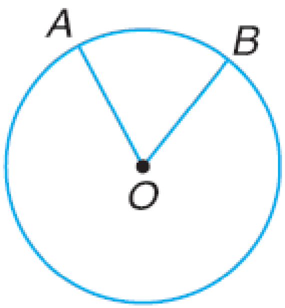
# Parts of a Circle

A circle is a special type of a geometric figure. All points on a **circle** are equidistant from a point called the center.




The measures of $\overline{OA}$ and $\overline{OB}$ are the same; that is, *OA* = *OB*.

Definition 1: A circle is the set of all points in a plane that are equidistant from a fixed point of the plane called the center of the circle.

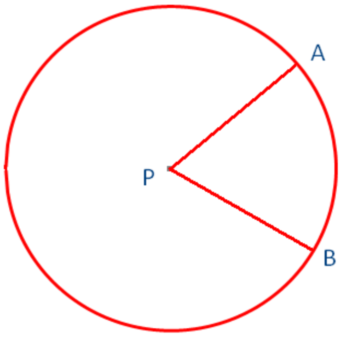
$P$ is the center

A $\in$ circle

PA is a radius

PA=PB

$\overline{PA} \cong \overline{PB}$

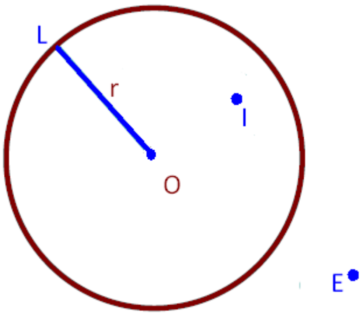



Note that a circle is named by its center. The circle above is named circle P.

Theorem 1: All radii of the same circle are congruent.

A circle separates a plane into three sets of points. The position of any point in the plane is determined by comparing the distance from this point to the center with the radius of the circle.

Point I is inside the circle since $OI < r$
Point E is outside the circle since $OE > r$
Point L is on the circle since $OL = r$

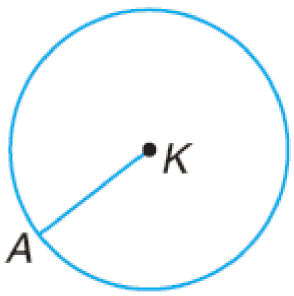


The **interior of a circle** is the set of all points whose distance from the center of the circle is less than the length of the radius of the circle.

The **exterior of a circle** is the set of all points whose distance from the center of the circle is greater than the length of the radius of the circle.

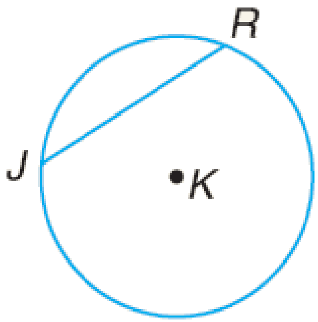
There are three kinds of segments related to circles. A **radius** is a segment whose endpoints are the center of the circle and a point on the circle. A **chord** is a segment whose endpoints are on the circle. A **diameter** is the longest chord; it passes through the center of the circle.

radius



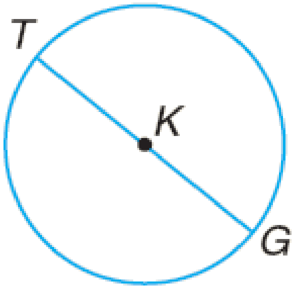

$\overline{KA}$ is a radius of ⊙ K

chord




$\overline{JR}$ is a chord of ⊙ K

diameter



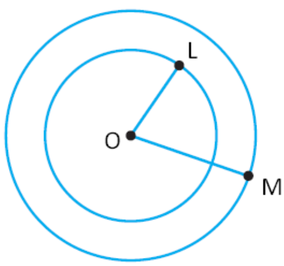

$\overline{TG}$ is a diameter of ⊙ K

Theorem 2: The measure of the diameter of a circle is twice the measure of the radius *r* of the circle.

Because all circles have the same shape, any two circles are similar. However, two circles are **congruent** if and only if their radii are congruent.

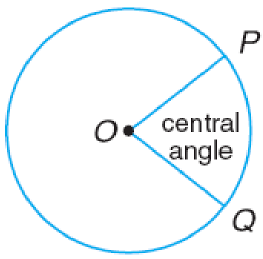
Two circles are **concentric** if they meet the following three requirements:

- They lie in the same plane.
- They have the same center.
- They have radii of different lengths.




The two circles $C(O,OL)$ and $C(O,OM)$ are concentric.

A **central angle** is formed when the two sides of an angle meet at the center of a circle. Each side intersects a point on the circle, dividing it into **arcs** that are curved lines.



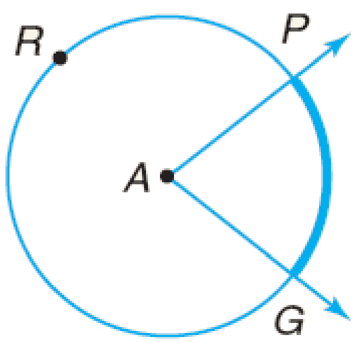
There are three types of arcs.
A **minor arc** is part of the circle in the interior of the central angle with measure less than 180°.
A **major arc** is part of the circle in the exterior of the central angle.
**Semicircles** are congruent arcs whose endpoints lie on a diameter of the circle.

Arcs are named by their endpoints. Besides the length of an arc, the measure of an arc is also related to the corresponding central angle.

Minor Arc Theorem: The degree measure of a minor arc is the degree measure of its central angle.

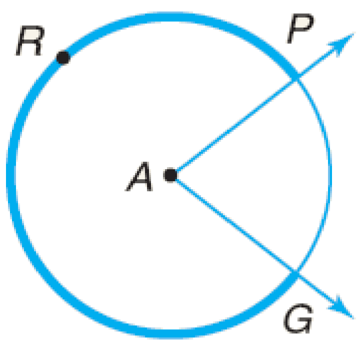
Major Arc Theorem: The degree measure of a major arc is 360 minus the degree measure of its central angle.

Definition of Semi – circle: The degree measure of a semicircle is 180.



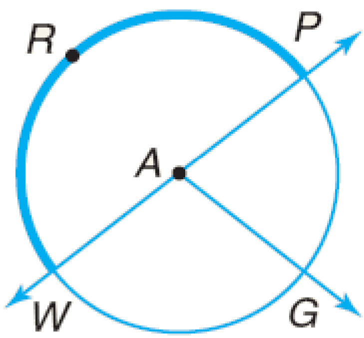

$PG$ is a minor arc

$mPG = m\angle PAG$




$PRG$ is a major arc

$mPRG = 360 - mPG$

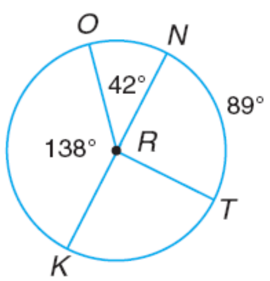



$PRW$ & $PGW$ are semicircles

$mPRW = mPGW = 180$

Note that for circle *A*, two letters are used to name the minor arc, but three letters are used to name the major arc and semicircle. These letters for naming arcs help us trace the set of points in the arc. In this way, there is no confusion about which arc is being considered.

Example 1: In $\odot R, \overline{KN}$ *is a diameter. Find* $mON, m\angle NRT, mOTK$ *and* $m\angle NTK$



$mON = m\angle ORN = 42^0$ Measure of minor arc

$m\angle NRT = mNT = 89^0$ Measure of central angle

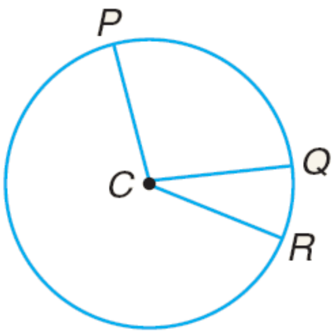
$mOTK = 360^0 - m\angle ORK = 360^0 - 138^0 = 222^0$ Measure of major arc

$mNTK = 180^0$ Measure of semicircle

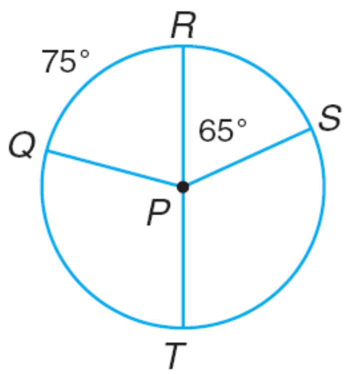
Postulate: **Arc Addition Postulate:** The sum of the measures of two adjacent arcs is the measure of the arc formed by the adjacent arcs.

If Q is a point on $PR$, then

$mPQ + mQR = mPQR$




Example 2: In $\odot P, \overline{RT}$ *is a diameter. Find* $mRS, mST, mSTR$ *and* $mQS$

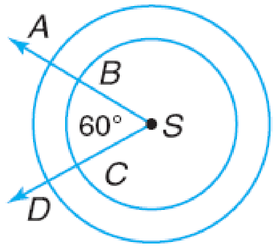



- $mRS = m\angle RPS = 65^0$ Measure of minor arc
- $mRS + mST = mRST$ Arc Addition Postulate

$\Rightarrow mST = mRST - mRS = 180^0 - 65^0 = 115^0$

- $mSTR = 360^0 - m\angle RPS = 360^0 - 65^0 = 295^0$ Measure of major arc
- $mQS = mQR + mRS = 75^0 + 65^0 = 140^0$ Arc Addition Postulate

Example 3: Suppose there are two concentric circles with $\angle ASD$ forming two minor arcs, $\overset{\frown}{BC}$ and $\overset{\frown}{AD}$. Are the two arcs congruent?



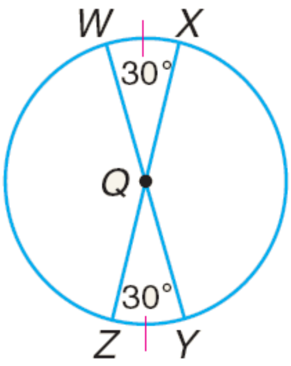

$m\overset{\frown}{BC} = m\angle BSC$ or $60^0$

$m\overset{\frown}{AD} = m\angle ASD$ or $60^0$

Although $\overset{\frown}{BC}$ and $\overset{\frown}{AD}$ each measure $60^0$, they are not congruent. The arcs are in circles with different radii, so they have different lengths. However, **in a circle, or in congruent circles, two arcs are congruent if they have the same measure.**

Central and Minor Arc Theorem: In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are congruent.

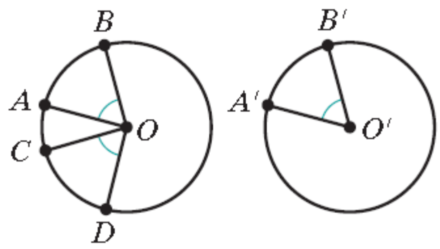
$$\overset{\frown}{WX} \cong \overset{\frown}{YZ} \Leftrightarrow m\angle WQX = m\angle YQZ$$




Example 4:

Given: $C(O,OA) \cong C'(O',O'A')$
$\angle AOB \cong \angle COD$
$\angle AOB \cong \angle A'O'B'$

Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ and $\overset{\frown}{AB} \cong \overset{\frown}{A'B'}$



Proof:

| Statements | Reasons |
|---|---|
| 1) $\angle AOB \cong \angle COD$ | 1) Given |
| 2) $\angle AOB \cong \angle A'O'B'$ | 2) Given |
| 3) $m\angle AOB = m\angle COD = m\angle A'O'B'$ | 3) Transitive property |
| 4) $\Rightarrow mAB = mCD = mA'B'$ | 4) Central angles are equal $\Rightarrow$ Arcs are equal |
| 5) $\Rightarrow AB \cong CD$ and $AB \cong A'B'$ | 5) Arcs having the same measure are congruent |

**Arc Length**

An arc is part of a circle, so arc length is part of the circumference.
The arc length is the measure of the distance along the curved line making up the arc. It is longer than the straight line distance between its endpoints (which would be a chord)

The formula for the arc measure is:

$$Arc\ \ Length = 2\pi R\left(\frac{C}{360}\right)$$

Where: C is the central angle of the arc in degrees
R is the radius of the arc
$\pi \approx 3.142$

Recall that 2πR is the circumference of the whole circle, so the formula simply reduces this by the ratio of the arc angle to a full angle ($360^0$).

Note: By transposing the above formula, you can solve for the radius, central angle, or arc length if you know any two of them.