Name: ____

Operations with Radicals

Radical expressions can be combined (added or subtracted) if they are like radicals – that is, they have the same index and radicand. For instance, $\sqrt{2}$, $3\sqrt{2}$ and $\frac{3}{2}\sqrt{2}$ are like radicals, but $\sqrt{2}$ and $\sqrt{3}$ are unlike radicals. To determine whether two radicals can be combined, you should first simplify each radical.

Rule 1: Let a and b be real numbers:

$$\sqrt{a+b}\ ^{1}\ \sqrt{a}+\sqrt{b} \qquad\qquad \sqrt{a}+\sqrt{a}=2\sqrt{a}$$

Rule 2: Let a , x and b be real numbers:

$$a\sqrt{x}+b\sqrt{x}=(a+b)\sqrt{x} \qquad\qquad a\sqrt{x}-b\sqrt{x}=(a-b)\sqrt{x}$$

Example 1: Combine the radicals and write your answer in simplest form

1) $4\sqrt{7}+11\sqrt{7}$

$4\sqrt{7}+11\sqrt{7}$

$=(4+11)\sqrt{7}$

$=15\sqrt{7}$

2) $2\sqrt{3}+6\sqrt{27}$

$2\sqrt{3}+6\sqrt{27}$

$=2\sqrt{3}+18\sqrt{3}$

$=(2+18)\sqrt{3}$

$=20\sqrt{3}$

3) $6\sqrt{9x}-3\sqrt{x}$

$6\sqrt{9x}-3\sqrt{x}$

$=18\sqrt{x}-3\sqrt{x}$

$=(18-3)\sqrt{x}$

$=15\sqrt{x}$

4) $\sqrt{x^5-x^4}-\sqrt{16x-16}$

$\sqrt{x^5-x^4}-\sqrt{16x-16}$

$=\sqrt{x^4(x-1)}-\sqrt{16(x-1)}$

$=x^2\sqrt{x-1}-4\sqrt{x-1}$

$=(x^2-4)\sqrt{x-1}$

$=(x-2)(x+2)\sqrt{x-1}$

Rule 3: The rules for doing arithmetic with square roots are quite simple:

1) Multiplication: "the product of square roots is the square root of the product"

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

When multiplying radicals, one must multiply the numbers OUTSIDE (O) the radicals AND then multiply the numbers INSIDE (I) the radicals.

$$O_1\sqrt{I_1} \bullet O_2\sqrt{I_2} = O_1 \bullet O_2\sqrt{I_1} \bullet \sqrt{I_2}$$

2) Division: "the quotient of square roots is the square root of the quotient"

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

When dividing radicals, one must divide the numbers OUTSIDE (O) the radicals AND then divide the numbers INSIDE (I) the radicals.

$$\frac{O_1\sqrt{I_1}}{O_2\sqrt{I_2}} = \frac{O_1}{O_2} \bullet \frac{\sqrt{I_1}}{\sqrt{I_2}}$$

Example 2: Simplify each expression:

1) $\sqrt{5} \times \sqrt{15}$

$$\sqrt{5} \times \sqrt{15} = \sqrt{5 \times 15} = \sqrt{75} = 5\sqrt{3}$$

2) $\frac{\sqrt{32}}{\sqrt{42}}$

$$\frac{\sqrt{32}}{\sqrt{42}} = \sqrt{\frac{32}{42}} = \sqrt{\frac{\cancel{32}^{16}}{\cancel{42}^{21}}} = \sqrt{\frac{16}{21}} = \frac{4}{\sqrt{21}}$$

3) $\sqrt{5}\left(3\sqrt{10} - 2\sqrt{5}\right)$

$$\sqrt{5}\left(3\sqrt{10} - 2\sqrt{5}\right)$$
$$= \left(\sqrt{5}\right)\left(3\sqrt{10}\right) - \left(\sqrt{5}\right)\left(2\sqrt{5}\right)$$
$$= 3\sqrt{50} - 2\sqrt{25}$$
$$= 15\sqrt{2} - 10$$

4) $\left(3+\sqrt{3}\right)\left(2-2\sqrt{3}\right)$

$\left(3+\sqrt{3}\right)\left(5-2\sqrt{3}\right)$

$=3\left(5-2\sqrt{3}\right)+\sqrt{3}\left(5-2\sqrt{3}\right)$

$=15-6\sqrt{3}+5\sqrt{3}-2\sqrt{9}$

$=15-\sqrt{3}-6$

$=9-\sqrt{3}$

Remark 1: $\sqrt{a^2b}=\sqrt{a^2}\cdot\sqrt{b}=a\sqrt{b}$

Example 3: Simplify

$$\frac{\sqrt[3]{40a^3b^{10}}}{\sqrt[4]{81a^4}}.$$

- 40 can be factored to 5×8 where 8 is a perfect cube.
- a^3 is a perfect cube.
- b^{10} can be factored to $b^9\times b$ where b^9 is a perfect cube.

$$\frac{\sqrt[3]{40a^3b^{10}}}{\sqrt[4]{81a^4}}$$

$$=\frac{\sqrt[3]{8\times 5\times a^3\times b^9\times b}}{\sqrt[4]{81\times a^4}}$$

$$=\frac{\sqrt[3]{8}\times\sqrt[3]{5}\times\sqrt[3]{a^3}\times\sqrt[3]{b^9}\times\sqrt[3]{b}}{\sqrt[4]{81}\times\sqrt[4]{a^4}}$$

$$=\frac{2\times\sqrt[3]{5}\times a\times b^3\times\sqrt[3]{b}}{3\times a}$$

$$=\frac{2ab^3\sqrt[3]{5b}}{3a}$$

$$=\frac{2b^3\sqrt[3]{5b}}{3}$$