Operations with Polynomial Functions

Rule 1: Adding and subtracting polynomials is simply the adding and subtracting of their like terms. There is a great similarity between the operations with polynomials and numbers. In subtraction the signs of all the subtracted terms must first be mentally changed and then the process completed as in addition.

Example 1: Given: $P(x)=3x^3+2x^2-4x+9$ and $Q(x)=5x^3-7x^2+7x-3$, find:

1) $P(x)+Q(x)$

$P(x)+Q(x)$ *Substitute the polynomials*

$=\left(3x^3+2x^2-4x+9\right)+\left(5x^3-7x^2+7x-3\right)$ *Remove the paranthesis*

$=3x^3+2x^2-4x+9+5x^3-7x^2+7x-3$ *Combine the like terms*

$=8x^3-5x^2+3x+6$

2) $P(x)-Q(x)$

$P(x)-Q(x)$ *Substitute the polynomials*

$=\left(3x^3+2x^2-4x+9\right)-\left(5x^3-7x^2+7x-3\right)$ *Remove the paranthesis*

$=3x^3+2x^2-4x+9-5x^3+7x^2-7x+3$ *Combine the like terms*

$=-2x^3+9x^2-11x+12$

Rule 2: The product of any two polynomials is found by multiplying each term of one by each term of the other and adding the results algebraically.

Example 2: Given: $P(x)=\sqrt{2}x^2-4\sqrt{3}x$ and $Q(x)=\sqrt{2}x-5\sqrt{3}$, find:

$P(x)\times Q(x)$

$P(x)\times Q(x)$ *Substitute the polynomials*

$=\left(\sqrt{2}x^2-4\sqrt{3}x\right)\left(\sqrt{2}x-5\sqrt{3}\right)$ *Multiplytheterms*

$=\left(\sqrt{2}x^2\right)\left(\sqrt{2}x\right)-\left(\sqrt{2}x^2\right)\left(5\sqrt{3}\right)-\left(4\sqrt{3}x\right)\left(\sqrt{2}x\right)+\left(4\sqrt{3}x\right)\left(5\sqrt{3}\right)$ *Simplify*

$=\sqrt{4}x^3-5\sqrt{6}x^2-4\sqrt{6}x^2+4\sqrt{9}x$ *Combine the like terms*

$=2x^3-9\sqrt{6}x^2+12x$

SPECIAL PRODUCTS

The products of certain polynomials occur frequently. It is convenient to remember the form of these products so that they can be written immediately without performing the complete multiplication process. We present seven of such special products as follows, and then show how each is derived:

1. **Product of the sum and difference of two numbers:** $a^2 - b^2 = (a - b)(a + b)$

$(4x^3 - \sqrt{2})(4x^3 + \sqrt{2})$ $\quad (a - b)(a + b) = a^2 - b^2$

$= (4x^3)^2 - (\sqrt{2})^2$ $\quad$ *Simplify*

$= 16x^6 - 2$

2. **Square the sum of two numbers:** $(a + b)^2 = a^2 + 2ab + b^2$

$(x^2 + 6)^2$ $\quad (a + b)^2 = a^2 + 2ab + b^2$

$= (x^2)^2 + 2(x^2)(6) + (6)^2$ $\quad$ *Simplify*

$= x^4 + 12x^2 - 36$

3. **Square of the difference of two numbers:** $(a - b)^2 = a^2 - 2ab + b^2$

$(2x^2 - 3)^2$ $\quad (a + b)^2 = a^2 + 2ab + b^2$

$= (2x^2)^2 - 2(2x^2)(3) + (3)^2$ $\quad$ *Simplify*

$= 4x^4 - 12x^2 + 9$

4. **Difference of two cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$(x^2 - 5)(x^4 + 5x^2 + 25)$

$= (x^2 - 5)((x^2)^2 + (5)(x^2) + (5)^2)$ $\quad (a - b)(a^2 + ab + b^2) = a^3 - b^3$

$= (x^2)^3 - (5)^3$ $\quad$ *Simplify*

$= x^6 - 125$

5. **Sum of two cubes:** $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$(x^2 + 3)(x^4 - 3x^2 + 9)$

$= (x^2 + 3)((x^2)^2 - (3)(x^2) + (3)^2)$ $\quad (a + b)(a^2 - ab + b^2) = a^3 + b^3$

$= (x^2)^3 - (3)^3$ $\quad$ *Simplify*

$= x^6 - 27$

6. **Cube of the difference of two numbers:** $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(y^3 - 2x)^3$ $\qquad (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= (y^3)^3 - 3(y^3)^2(2x) + 3(y^3)(2x)^2 - (2x)^3$ *Simplify*

$= y^9 - 6y^6x + 12x^2y^3 - 4x^3$

7. **Cube of the sum of two numbers:** $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(y^2 + \sqrt{2}z^2)^3$ $\qquad (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= (y^2)^3 - 3(y^2)^2(\sqrt{2}z^2) + 3(y^2)(\sqrt{2}z^2)^2 - (\sqrt{2}z^2)^3$ *Simplify*

$= y^6 - 3\sqrt{2}y^4z^2 + 6y^2z^4 - 2\sqrt{2}z^6$