Operations with Complex Numbers

A complex number is of the form $a+bi$, where a is called the real part and bi is called the imaginary part. When performing operations involving complex numbers, we will be able to use many of the techniques we used with polynomials.

Rule 1: **Addition and Subtraction of Complex Numbers**

When adding and subtracting complex numbers, we are **only allowed** to add real parts to other real parts, and imaginary parts to other imaginary parts.

Addition of two complex numbers a + b i and c + d i is defined as follows:

(a + b i) + (c + d i) = (a + c) + (b + d) i

The subtraction of two complex numbers a + b i and c + d i is defined as follows:

(a + b i) - (c + d i) = (a - c) + (b - d) i

Example 1: Find the sum: $(7+3i)+(5+2i)$

Add the real parts together.

$7+5=12$

Add the imaginary parts together.

$3i+2i=5i$

The solution is: $12+5i$

Rule 2: **Multiplication of Complex Numbers**

Multiplying complex numbers works like multiplying two binomials.

The multiplication of two complex numbers a + b i and c + d i is defined as follows.

(a + b i)(c + d i) = (a c - b d) + (a d + bc) i

Example 2: Find the product $(3+2i)(2-4i)$

$(3+2i)(2-4i)$

$=(3)(2)-(3)(4i)+(2i)(2)-(2i)(4i)$

$=6-12i+4i-8i^2$

$=6-8i-8(-1)$

$=6-8i+8$

$=14-8i$

Division of Complex Numbers

We use the multiplication property of a complex number and its conjugate to divide two complex numbers. Earlier, we learned how to rationalize the denominator of an expression like: $\frac{3}{2+\sqrt{5}}$

We multiplied numerator and denominator by the conjugate of the denominator, $2+\sqrt{5}$:

$$\frac{3}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}=\frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}=\frac{6-3\sqrt{5}}{4-5}=-1\left(6-3\sqrt{5}\right)=3\sqrt{5}-6$$

We did this so that we would be left with no radical (square root) in the denominator. Dividing complex numbers is similar. The division of two complex numbers, $\frac{z_1}{z_2}=\frac{a+bi}{c+di}$ can be thought of as simply a process for eliminating the *i* from the denominator and writing the result as a new complex number $a+bi$

$$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\bullet\frac{c-di}{c-di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{ac-adi+bci-bdi^2}{c^2-d^2i^2}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$$

Example 3: Write $\frac{2-i}{5+3i}$ in the form of $a+bi$.

The **conjugate** of $5+3i$ is $5-3i$

$$\frac{2-i}{5+3i}$$
$$=\frac{2-i}{5+3i}\times\frac{5-3i}{5-3i}$$
$$=\frac{(2-i)(5-3i)}{(5+3i)(5-3i)}$$
$$=\frac{10-6i-5i+3i^2}{25-9i^2}$$
$$=\frac{10-11i+3i^2}{25-9i^2}$$
$$=\frac{10-11i-3}{25+9}$$
$$=\frac{7-11i}{34}$$
$$=\frac{7}{34}-\frac{11}{34}i$$