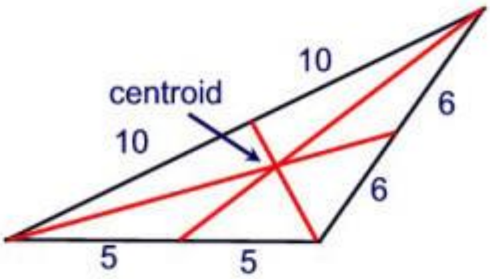
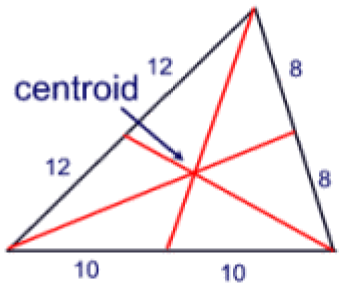
Medians and Altitudes of a Triangle

Definition 1: **Median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

Definition 2: **Centroid of the triangle** is the point of concurrency of the three medians of a triangle.



Medians of an obtuse triangle.



Medians of an acute triangle.

Notice that the point of concurrence is in the interior of the triangles.

Theorem 1: **Concurrency of Medians of a Triangle**: the medians of a triangle intersect at a point that is called the Centroid and that is two thirds of the distance from each vertex to the midpoint of the opposite side.

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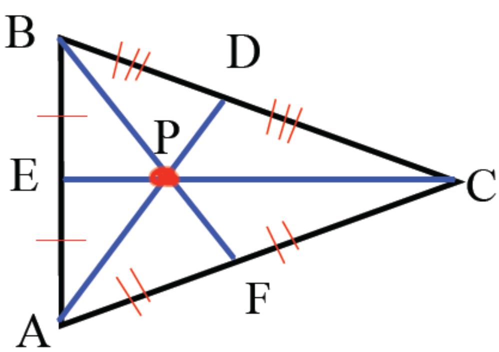
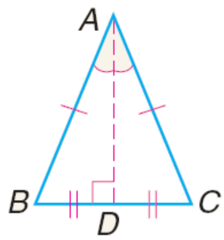
If point P is the Centroid of ΔABC, then $AP = \frac{2}{3}AD$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CE$



Do you know? Archimedes showed that the point where the medians are concurrent is the center of gravity of a triangular shape of uniform thickness and density. This point where the medians are concurrent is called the **Centroid** of a triangle. If you cut a triangle out of cardboard and balance it on a pointed object, such as a pencil, the pencil will mark the location of the triangle's Centroid. The Centroid divides the medians into a 2:1 ratio. The section of the median nearest the vertex is twice as long as the section nears the midpoint of the triangle's side.

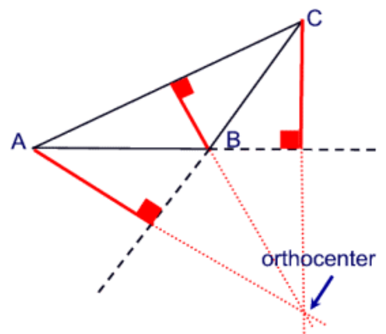
Theorem 2: The median from the vertex angle of an isosceles triangle lies on the perpendicular bisector of the base and the angle bisector of the vertex angle.



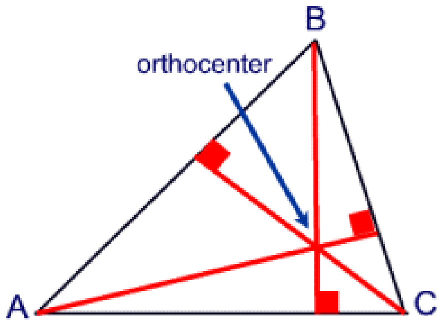
Definition 3: **Altitude of the triangle**: is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side. An altitude can lie inside, on, or outside the triangle.

Definition 4: **Orthocenter of the triangle:** is the point of concurrency of the three altitudes of a triangle. (The prefix "ortho" means "right".)

Theorem 3: **Concurrency of Altitudes of a Triangle**: the lines containing the altitudes of a triangle are concurrent at the orthocenter.



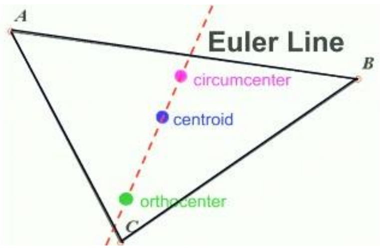
Altitudes of an obtuse triangle.



Altitudes of an acute triangle.

<u>**Notice that the point of concurrence is not necessarily inside the triangle.**</u>

Euler Line: In any triangle, the circumcenter, Centroid, and orthocenter are collinear (lie on the same straight line). The collinear line upon which these three points lie is called the Euler line. The Centroid is always located between the circumcenter and the orthocenter. The Centroid is twice as close to the circumcenter as to the orthocenter.



The word "Euler" is pronounced as if it were spelled "Oiler" and refers to the mathematician Leonhard Euler (1707-1783).

Definition 5: The line through H, O, and G is called the Euler line of the triangle.

Theorem 4: **Euler Line Theorem:** The orthocenter H, the circumcenter O, and the centroid G of any triangle are collinear. Furthermore, G is between H and O (unless the triangle is equilateral, in which case the three points coincide).