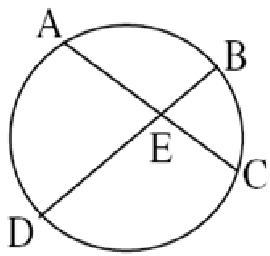
Measures of Tangent, Chords, and Secant Segments

Segments Formed by Two Intersecting Chords

Length of Inscribed Cords Theorem: If two chords intersect within a circle, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other.

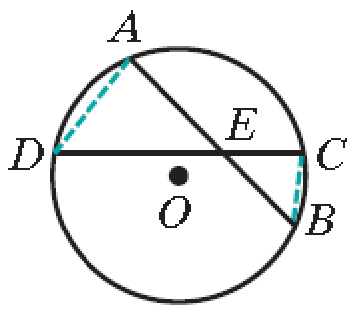
$$(AE)(EC)=(DE)(EB)$$



Example 1:

Given: Chords AB and CD intersects at E in the interior of the circle of center O

Prove: $(AE)(EB)=(CE)(ED)$



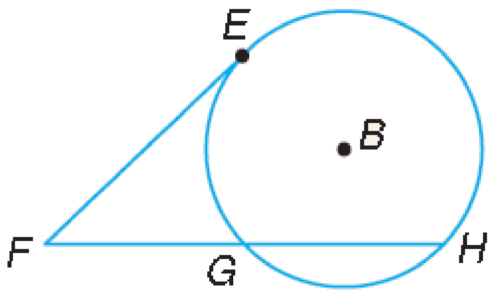
Proof:

Statements	Reasons
Join A and D, C and B	
1) $\angle A \cong \angle C; \angle D \cong \angle B$	1) Inscribed angles of a circle that intercept the same arc are congruent
2) $\triangle ADE \cong \triangle CBE$	2) AA similarity theorem
3) $\dfrac{AE}{CE}=\dfrac{ED}{EB}$	3) The lengths of the corresponding sides of similar triangles are in proportion
4) $(AE)(EB)=(CE)(ED)$	4) In a proportion, the product of the means is equal to the product of the extremes

Segments Formed by a Tangent Intersecting a Secant

Theorem 1: If a tangent and a secant are drawn to a circle from an external point, then the square of the length of the tangent segment is equal to the product of the lengths of the secant segment and its external segment.

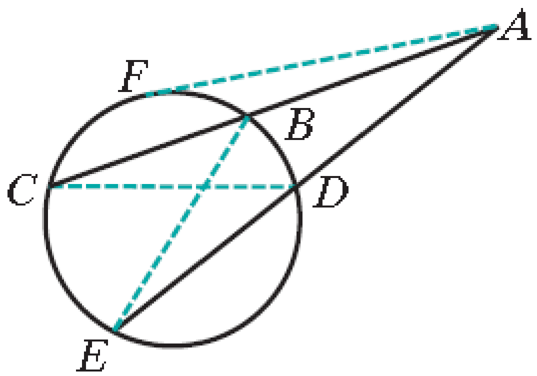
$$(FE)^2 = FH \bullet FG$$



Theorem 2: If a tangent and a secant are drawn to a circle from an external point, then the length of the tangent segment is the mean proportional between the lengths of the secant segment and its external segment.

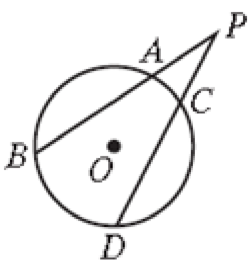
What is the relationship between the lengths of the two secants to a circle from an external point? Let $\overline{ABC}$ and $\overline{ADE}$ be two secant segments drawn to a circle as shown in the diagram.

Draw $\overline{AF}$ a tangent segment to the circle from A. Since: $(AF)^2 = AC \bullet AB$ and $(AF)^2 = AE \bullet AD$ then, $AE \bullet AD = AC \bullet AB$



Theorem 3: If two secant segments are drawn to a circle from an external point, then the product of the lengths of one secant segment and its external segment is equal to the product of the lengths of the other secant segment and its external segment.

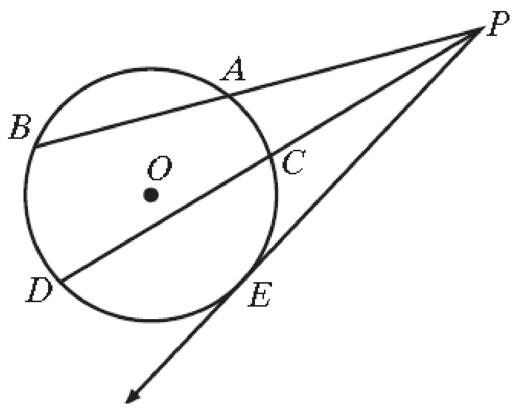
$$(PB)(PA) = (PD)(PC)$$



Example 2: Two secant segments, $\overline{PAB}$ and $\overline{PCD}$, and a tangent segment, $\overline{PE}$, are drawn to a circle from an external point P. If PB=9 cm, PD=12 cm, and the external segment of $\overline{PAB}$ is 1 cm longer than the external segment of $\overline{PCD}$, find: PA, PC, and PE

Let x=PC and PA=x+1

$$(PB)(PA)=(PD)(PC)$$
$$9PA=12PC$$
$$9(x+1)=12x$$
$$9x+9=12x$$
$$3x=9$$
$$x=3$$

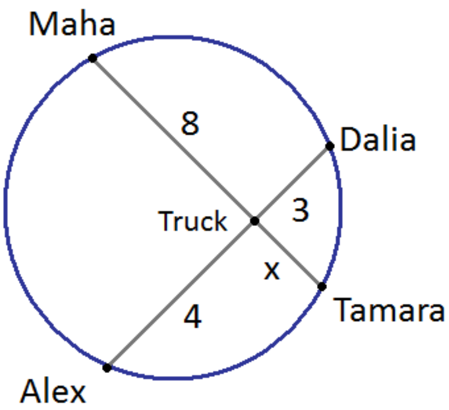


$$\Rightarrow PA=x+1=3+1=4$$
$$\Rightarrow PC=x=3$$

$$\Rightarrow (PE)^2=PB\bullet PA$$
$$\Rightarrow (PE)^2=9\bullet 4$$
$$\Rightarrow (PE)^2=36$$
$$Therefore, PE=6$$

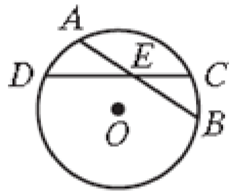
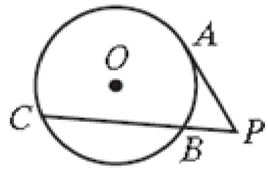
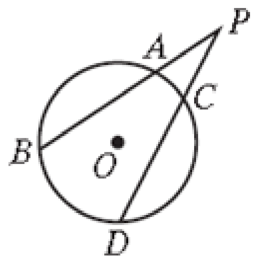
Example 3: A toy truck is located within a circular play area. Alex and Dalia are sitting on opposite endpoints of a chord that contains the truck. Alex is 4 feet from the truck, and Dalia is 3 feet from the truck. Maha and Tamara are sitting on opposite endpoints of another chord containing the truck. Maha is 8 feet from the truck. How many feet is Tamara from the truck?

Use the Length of Inscribed Chords Theorem to solve.



$$pq=rs$$
$$8x=3\times 4$$
$$8x=12$$
$$x=\boxed{1.5}\text{ feet}$$

Check the summary table below:

Type of Segment	Length	Example
Formed by Two Intersecting Chords	If two chords intersect, the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other	 $(AE)(EB)=(CE)(ED)$
Formed by a Tangent Intersecting a Secant	If a tangent and a secant are drawn to a circle from an external point, then the square of the length of the tangent segment is equal to the lengths of the secant segment and its external segment.	 $(PA)^2 = PC \bullet PB$
Formed by Two Intersecting Secants	It two secant segments are drawn to a circle from an external point, then the product of the lengths of one secant segment and its external segment is equal to the product of the lengths of the other secant segment and its external segment.	 $(PB)(PA)=(PD)(PC)$