Matrices

An automobile dealer sells four different models whose fuel economy is shown below

	Sports car	Sedan	Station wagon	Van
Miles per gallon for city driving	18	22	18	15
Miles per gallon for highway driving	22	31	25	19

The information in the table can be displayed as a rectangular array of numbers enclosed by brackets. Such an array is called a matrix (plural matrices). A matrix is usually named by a capital letter. In this example, M is used to name the matrix.

$$M = \begin{bmatrix} 18 & 22 & 18 & 15 \\ 22 & 31 & 25 & 19 \end{bmatrix}$$

Each number in a matrix is an **element** (or **entry**) of the matrix. The **dimensions** of a matrix are the number of rows and columns.
A *matrix* is simply a rectangular table of numbers written in either ( ) or [ ] brackets.
The **size** of a matrix is written: rows × columns.

This is a 2 × 4 matrix: $\begin{pmatrix} 2 & 4 & -1 & 0 \\ 1 & 3 & 7 & 2 \end{pmatrix}$	This is a 4 × 1 matrix. $\begin{bmatrix} 4 \\ 3.1 \\ -8.3 \\ 7.9 \end{bmatrix}$	This is a 3 × 3 matrix. A matrix with the same number of rows and columns is called a **square** matrix. $\begin{pmatrix} 2 & 4 & 0 \\ 1 & 3 & 2 \\ 3 & -4 & -9 \end{pmatrix}$

Definition 1: Elements in a matrix
The **elements in a matrix *A*** are denoted by a_{ij}, where *i* is the row number and *j* is the column number. So the general form is:

$$\begin{pmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{pmatrix}$$

Example 1: Consider the matrix

$$A=\begin{pmatrix} 7 & 4 & -1 \\ 1 & 3 & 7 \end{pmatrix}$$

What is:

1) a_{21}

The element a_{21} = 1, since the element in the 2nd row and 1st column is 1.

2) a_{13}

The element a_{13} = -1, since the element in the 1st row and 3rd column is -1.

Definition 2: Equality of Matrices

Two matrices are equal if and only if they have the **same dimensions**, and the elements in all corresponding positions (same row, same column) are equal.

Equal matrices have identical corresponding elements.

Matrices of different dimensions can never be equal.

Example 2: If $\begin{pmatrix} 1 & x \\ y & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 7 & a \end{pmatrix}$ then find the values of x, y and z.

x = 2, y = 7 and a = 3

Definition 3: A matrix A can be transposed into another matrix A^t by interchanging the rows and columns of A.

Example 3: Given the following matrix, find the transpose of A.

$$A=\begin{bmatrix} 17 & 21 & 6 & -9 \\ 23 & 30 & 4 & 5 \end{bmatrix}, \qquad A^t=\begin{bmatrix} 17 & 23 \\ 21 & 30 \\ 6 & 4 \\ -9 & 5 \end{bmatrix}$$

The rows of A^t are the same as the columns of A. The dimensions of A^t is 4×2 while the dimensions of A are 2×4

Definition 4: **Addition (and Subtraction) of Matrices**

We can only add (or subtract) matrices if they have the same dimensions. That is, the two matrices must have the same number of rows and the same number of columns.

Adding two matrices will produce a new matrix by finding the sums of the corresponding elements of the matrices.

Two matrices cannot be added if they have different dimensions.

Example 4: Perform the indicated operation

$$\begin{pmatrix} 6 & -5 & 4 \\ 0 & 7 & 5 \end{pmatrix} + \begin{pmatrix} 3 & -2 & 7 \\ -5 & 5 & 9 \end{pmatrix} = \begin{pmatrix} 6+3 & -5+(-2) & 4+7 \\ -5+0 & 7+5 & 5+9 \end{pmatrix} = \begin{pmatrix} 9 & -7 & 11 \\ -5 & 12 & 14 \end{pmatrix}$$

Matrix subtraction is similar to real number subtraction: to subtract a matrix, we add the additive inverse of the matrix. More simply, two matrices can be subtracted by finding the differences of the corresponding elements of the matrices.

Example 5: Perform the indicated operation

$$\begin{pmatrix} 6 & -5 & 4 \\ 0 & 7 & 5 \end{pmatrix} + \begin{pmatrix} 3 & -2 & 7 \\ -5 & 5 & 9 \end{pmatrix} = \begin{pmatrix} 6-3 & -5-(-2) & 4-7 \\ 0-(-5) & 7-5 & 5-9 \end{pmatrix} = \begin{pmatrix} 3 & -3 & -3 \\ 5 & 2 & -4 \end{pmatrix}$$

Definition 5: **Scalar Multiplication (and Division)**
Scalar multiplication of matrices is similar to scalar multiplication of **vectors**.
We multiply (or divide) each element by the scalar value (a single number).

Example 6: If $A = \begin{pmatrix} 3 & 1 \\ 7 & -1 \\ 2 & 8 \end{pmatrix}$, then find $3A$

$$3A = 3\begin{pmatrix} 3 & 1 \\ 7 & -1 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 3\times3 & 1\times3 \\ 7\times3 & -1\times3 \\ 2\times3 & 8\times3 \end{pmatrix} = \begin{pmatrix} 9 & 3 \\ 21 & -3 \\ 6 & 24 \end{pmatrix}$$

When doing scalar multiplication, if we start with a 3 × 2 matrix, we end with a 3 × 2 matrix.