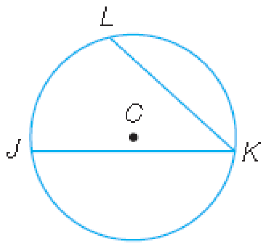
Inscribed Angles and Their Measures

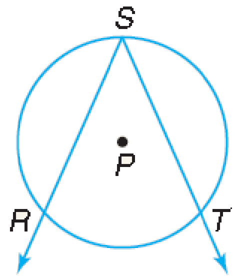


Recall that a polygon can be inscribed in a circle. An angle can also be inscribed in a circle. An inscribed angle is an angle whose vertex is on the circle and whose sides contain chords of the circle. Angle *JKL* is an inscribed angle.

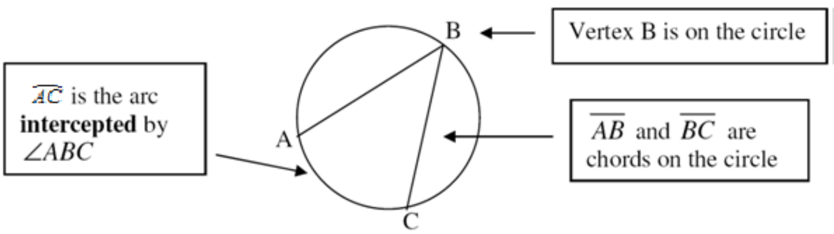
Notice that *K*, the vertex of $\angle JKL$, lies on the circle of *C*. The sides of $\angle JKL$ contain chords *LK* and *JK*. Therefore, $\angle JKL$ is an inscribed angle. Each side of the inscribed angle intersects the circle at a point. The two points *J* and *L* form an arc. We say that $\angle JKL$ *intercepts* JL, or that JL is the intercepted arc of $\angle JKL$.

Definition 1: An angle is inscribed if and only if its vertex lies on the circle and its sides contain chords of the circle.

$\angle RST$ is an inscribed angle because S belongs to the circle and $\overline{SR}$ and $\overline{ST}$



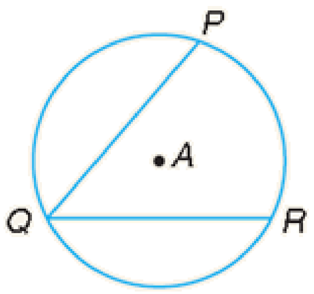
The diagram below shows the different parts that are related to inscribed angle.



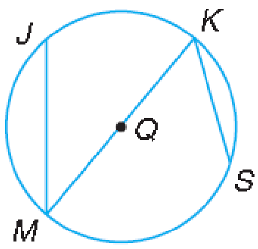
Theorem 1: **Inscribed Angle Theorem:** If an angle is inscribed in a circle, then its measure equals one-half the measure of its intercepted arc (or the measure of the intercepted arc is twice the measure of the inscribed angle).

$$m\angle PQR = \frac{1}{2}(mPR)$$

$$2(m\angle PQR) = mPR$$



Example 1: Refer to the diagram below to find the required measures:



1) If $mJK = 80^0$, find $m\angle JMK$.

   $m\angle JMK = \frac{1}{2}\left(mJK\right) = \frac{1}{2}\left(80^0\right) = 40^0$ (Inscribed angle theorem)

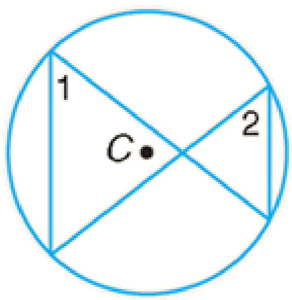
2) If $m\angle MKS = 56^0$, find mMS.

   $mMS = 2\left(m\angle MKS\right) = 2\left(56^0\right) = 112^0$ (Inscribed angle theorem)

Arc – Intercept Corollary: If two inscribed angles of a circle (or congruent circles) intercept congruent arcs or the same arc, then the angles are congruent.

Inscribed $\angle$'s of $\cong$ arcs are $\cong$.
Inscribed $\angle$'s of same arc are $\cong$.

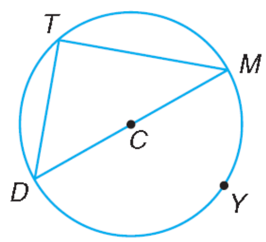
$\angle 1 \cong \angle 2$



Suppose $\angle MTD$ is inscribed in $\odot C$ and intercepts semicircle MYD.

Since $mMYD = 180^0$, $m\angle MTD = \frac{1}{2} \bullet 180^0 = 90^0$.

Therefore, $\angle MTD$ is a right angle.

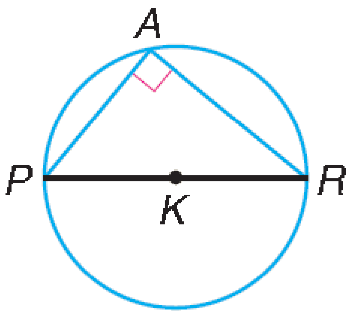


Right Angle Corollary: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.

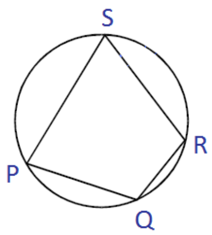
PR is a semicircle

$\Rightarrow m\,PR = 180^0$

$\Rightarrow m\angle PAR = 90^0$

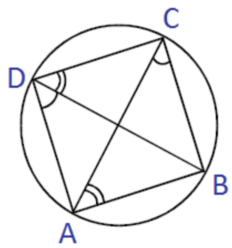


Definition 2: A quadrilateral is said to be a cyclic quadrilateral if there is a circle passing through all its four vertices.



Cyclic Quadrilateral Theorem: If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

Example 2:



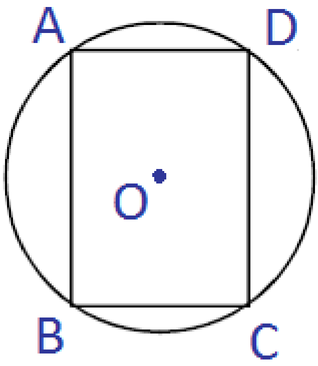
Given: A cyclic quadrilateral ABCD

Prove: $m\angle BAD + m\angle BCD = m\angle ABC + m\angle ADC = 180^\circ$

Statements	Reasons
1) $m\angle ACB = m\angle ADB$	1) Angles in the same segment
2) $m\angle BAC = m\angle BDC$	2) Angles in the same segment
3) $m\angle ACB + m\angle BAC = m\angle ADB + m\angle BDC$	3) Addition postulate
But $\angle ADB$ and $\angle BDC$ are adjacent angles	
4) $\angle ADB + \angle BDC = \angle ADC$	4)Angle Sum theorem
5) $m\angle ACB + m\angle BAC = m\angle ADC$	5)Substitution
6) $m\angle ACB + m\angle BAC + m\angle ABC = m\angle ADC + m\angle ABC$	6)Adding $\angle ABC$ to both sides
7) $m\angle ACB + m\angle BAC + m\angle ABC = 180^\circ$	7) Triangle Sum Theorem
8) $m\angle ADC + m\angle ABC = 180^\circ$	8)Substitution
9) $m\angle BAD + m\angle BCD + m\angle ADC + m\angle ABC = 360^\circ$	9)Interior Angles theorem
10) $m\angle BAD + m\angle BCD = 360^\circ - (m\angle ADC + m\angle ABC)$ $m\angle BAD + m\angle BCD = 180^\circ$	

Converse of Cyclic Quadrilateral Theorem: If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

Example 3: ABCD is a cyclic parallelogram. Show that it is a rectangle.



Given: ABCD is a cyclic parallelogram

Prove: ABCD is a rectangle

Proof:

Statements	Reasons
1) $m\angle BAD + m\angle BCD = 180^\circ$	1) ABCD is a cyclic quadrilateral
2) $\angle BAD \cong \angle BCD$	2) Opposite angles of a parm
3) $m\angle BAD + m\angle BAD = 180^\circ$	3) Substitution
4) $2m\angle BAD = 180^\circ$	
5) $m\angle BAD = 90^\circ$	
6) ABCD is a rectangle	6) A parm with one right angle is a rectangle