TImaginary and Complex Numbers

A positive number squared or a negative number squared will always equal a positive number. We know how to calculate the square roots of positive numbers but what about the square roots of negative numbers. How can we find their values?
So, "Does this mean that there isn't a solution to the equation?" Although the real-number system did not contain the solution to the equation, a solution really does exist. So, another set of numbers to describe these solutions have been developed. This new set of numbers is called the **imaginary numbers** and the combination of these imaginary numbers with the real numbers is called the **complex number system**.

Mathematicians have designated a special number 'i' which is equal to the square root of minus 1. Then, it follows that $i^2 = -1$. This new number is called "*i*", standing for "imaginary"

Definition 1: Imaginary number: $i^2 = -1 \Leftrightarrow i = \sqrt{-1}$

Example 1: Simplify:

1) $\sqrt{-16}$ $\qquad \sqrt{-16} = \sqrt{(-1)(16)} = \sqrt{(i^2)(4^2)} = \sqrt{(4i)^2} = 4i$

2) $\sqrt{-144}$ $\qquad \sqrt{-144} = \sqrt{(-1)(144)} = \sqrt{(i^2)(12^2)} = \sqrt{(12i)^2} = 12i$

Let us check the table to notice the relation between the results of even and odd powers of the imaginary number i.

$i = \sqrt{-1}$ $\qquad i^2 = -1$

$i^3 = i^2 \bullet i = (-1)i = -i$ $\qquad i^4 = i^2 \bullet i^2 = (-1)(-1) = 1$

$i^5 = i^2 \bullet i^2 \bullet i = (-1)(-1)i = i$ $\qquad i^6 = i^2 \bullet i^2 \bullet i^2 = (i^3)^2 = (-i)^2 = i^2 = -1$

$i^7 = (i^2)^3 \bullet i = (-1)^3 i = -i$ $\qquad i^8 = (i^2)^4 = (-1)^4 = 1$

$i^9 = (i^2)^4 \bullet i = (-1)^4 i = i$ $\qquad i^{10} = (i^2)^5 = (-1)^5 = -1$

$i^{11} = (i^2)^5 \bullet i = (-1)^5 i = -i$ $\qquad i^{12} = (i^2)^6 = (-1)^6 = 1$

<u>Powers if *i*</u>

To determine the values of *i* raised to a power, divide the power by 4.

- If the remainder is 1, the value of the expression is *i.*
- If the remainder is 2, the value of the expression is – 1.
- If the remainder is 3, the value of the expression is – *i.*
- If the remainer is 0, the value of the expression is 1.

Example 2: Simplify

1) i^{97} $\quad i^{97} = i^{96+1} = i^{4(24)+1} = (i^4)^{24} \bullet i = (1)^{24} \bullet i = (1) \bullet i$

2) i^{19} $\quad i^{19} = i^{16+3} = i^{4(4)+1} = (i^4)^4 \bullet i^3 = (1)^4 \bullet i^3 = (1) \bullet -i = -i$

Definition 2: A **complex number** is a number which can be written in the standard form $a + bi$, where *a* and *b* are real numbers. We call *a* the real part and *bi* the imaginary part.

- If $b = 0$, then $a + 0i = a$ is a real number.
- If $a = 0$ and $b \neq 0$, then $0 + bi = bi$ is a pure imaginary number.
- If $b \neq 0$, then $a + bi$ is complex number.

The "standard" form of a complex numbers is "$a + bi$"